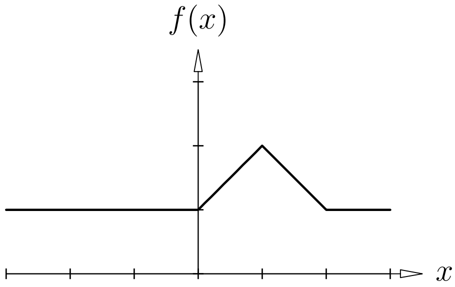
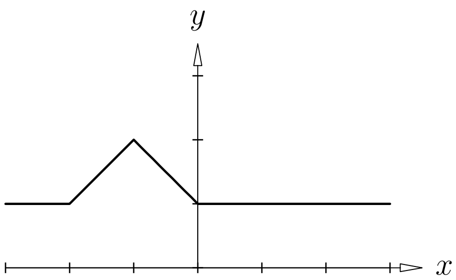
College of the Holy Cross, Fall Semester, 2005
Math 131, Midterm 1 Solutions

1. [16 points] The graph of a function f is shown below.

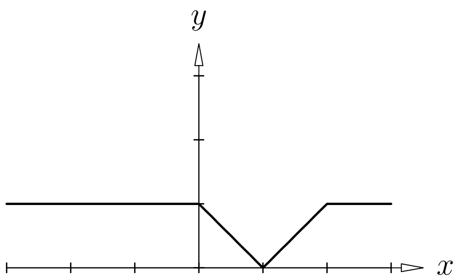


Each figure below represents the graph of one of the formulas (a)-(f). Match each graph with its formula. The marks on the axes are spaced 1 unit apart.

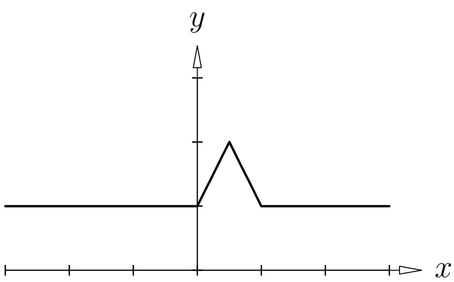
(I) <u>f</u>



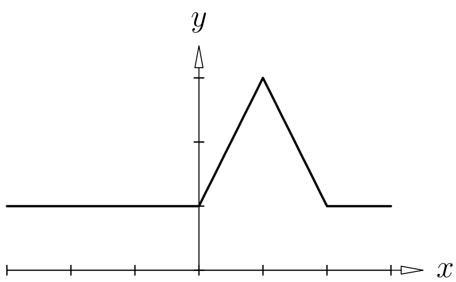
(II) <u>b</u>



(III) <u>d</u>



(IV) <u>a</u>



(a) $2f(x) - 1$

(b) $2 - f(x)$

(c) $f(\frac{1}{2}x)$

(d) $f(2x)$

(e) $f(x - 2)$

(f) $f(x + 2)$

2. Consider the function $r(t) = 3\sin(\pi t) + 2$.

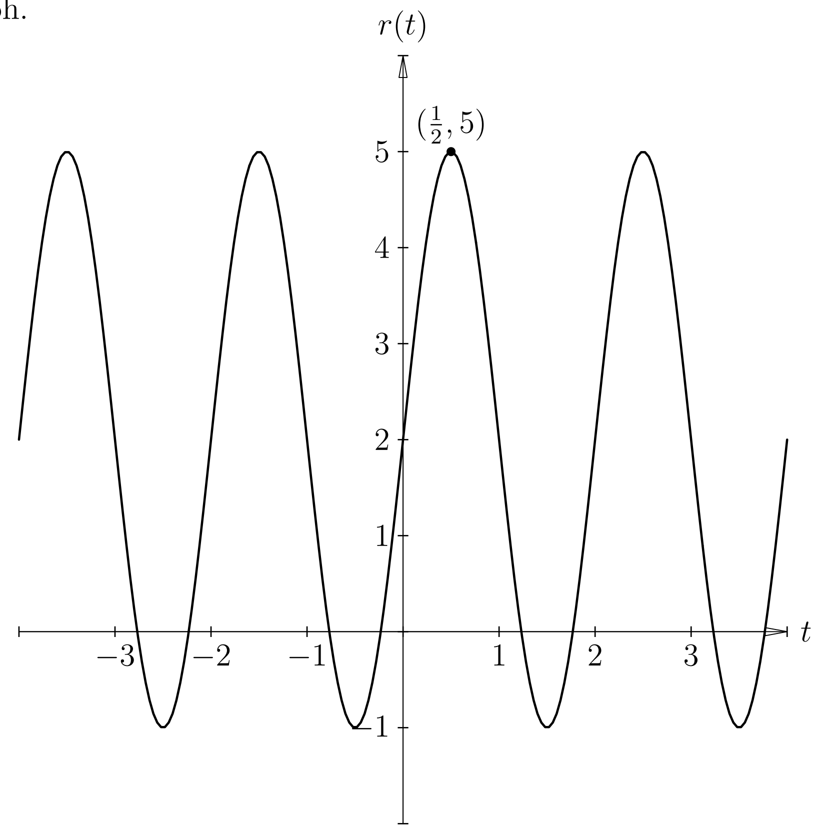
   (a) [4 points] Find the period of r.

   The period is $\dfrac{2\pi}{\pi} = 2$

   (b) [4 points] Find the amplitude of r.

   The amplitude is 3.

   (c) [6 points] Sketch the graph of $r(t)$. Label the coordinates of one of the "peaks" of the graph.



3. Consider the function

$$f(x) = \frac{4x^2 - 16}{x^2}$$

   (a) [5 points] Find the zeros (roots) of f.

   $$f(x) = \frac{4(x^2 - 4)}{x^2} = \frac{4(x-2)(x+2)}{x^2}$$

   Thus, $f(x) = 0$ if $x = 2$ or $x = -2$.

(b) [5 points] Does f have horizontal asymptotes? If yes, give the asymptotes (i.e. find all values of k such that $y = k$ is a horizontal asymptote).

$\lim_{x\to\infty} \frac{4x^2 - 16}{x^2} = 4$ and $\lim_{x\to-\infty} \frac{4x^2 - 16}{x^2} = 4$.

Thus $y = 4$ is a horizontal asymptote.

(c) [5 points] Does f have vertical asymptotes? If yes, give the asymptotes (i.e. find all values of l such that $x = l$ is a vertical asymptote).

Since $f(x)$ is not defined at $x = 0$, consider $\lim_{x\to 0} \frac{4x^2 - 16}{x^2} = -\infty$.

Thus $x = 0$ is a vertical asymptote.

4. [8 points] Suppose that $C(t)$ is a function measuring the cost C of a bleacher seat at Fenway Park in the year t, where $t = 0$ corresponds to the year 1950. Explain in words the meaning of $C^{-1}(3) = 30$.

In 1980 the cost of a bleacher seat was $3.

5. [14 points] The number of bacteria in milk grows at a rate of 10% per day once the milk has been bottled. You purchase a bottle with 50,000 BPM (bacteria per milliliter). Suppose that milk is unsafe to drink once the bacteria count reaches 210,000 BPM. How many days until your milk goes bad? Give both an **exact answer** and a numerical answer rounded to two decimal places.

The number of bacteria at time t is given by $P(t) = P_0 a^t$, where $a = 1.1$ and $P_0 = 50,000$.

Thus, $P(t) = 50,000(1.1)^t$.

Find t for which $50,000(1.1)^t = 210,000$. This means $(1.1)^t = \frac{21}{5}$.

So, $t \ln(1.1) = \ln \frac{21}{5}$. Therefore, $t = \frac{\ln 4.2}{\ln 1.1} \approx \frac{1.4351}{0.0953} \approx 15.05$ days.

6. The table below shows the entries for a linear function, an exponential function and a quadratic power function (a function of the form kx^2). Note that the x entries increase by 2 not 1.

x	$f(x)$	$g(x)$	$h(x)$
0	0	16	16
2	1	8	4
4	4	0	1
6	9	-8	0.25
8			

(a) [6 points] Match each of the functions f, g or h with its correct type (linear, exponential, quadratic). Be sure to explain your answer thoroughly.

The difference in the values of g is constant for equally spaced values of x: $g(2) - g(0) = g(4) - g(2) = g(6) - g(4) = -8$. So g is linear.

The quotient of values of h is constant for equally spaced values of x: $\frac{h(2)}{h(0)} = \frac{h(4)}{h(2)} = \frac{h(6)}{h(4)} = \frac{1}{4}$. So h is exponential.

If f is quadratic, it would be of the form $f(x) = kx^2$. Since $f(2) = 1$, we would have $k2^2 = 1$, which implies $k = \frac{1}{4}$. Then the formula for f would be $f(x) = \frac{1}{4}x^2$. And indeed we would have $f(0) = 0$, $f(4) = 4$ and $f(6) = 9$. So f is quadratic.

(b) [9 points] Find formulas for each function $f(x), g(x)$ and $h(x)$.

From above $f(x) = \frac{1}{4}x^2$.

$g(x) = mx + b$. Since $g(0) = 16$, we have $b = 16$. Since $\frac{g(2) - g(0)}{2 - 0} = \frac{-8}{2} = -8$, we have $m = -8$. Therefore $g(x) = -4x + 16$.

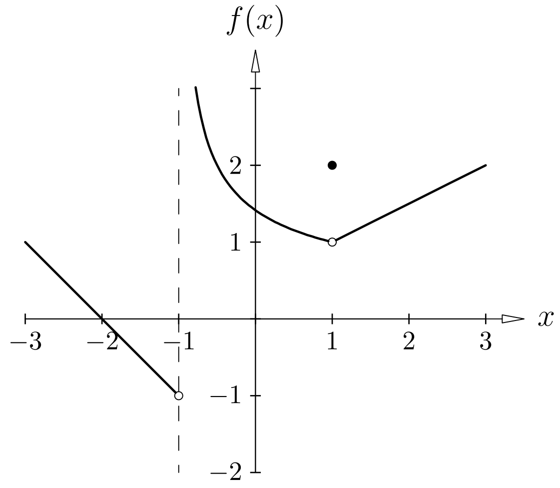
$h(x) = H_0 a^x$. Since $h(0) = 16$, we have $H_0 = 16$ and $h(x) = 16a^x$. Since $h(2) = 4$, we have $16a^2 = 4$, which implies $a^2 = 1/4$ or $a = 1/2$ (recall that exponential functions do not make sense for negative a). Therefore $h(x) = 16(1/2)^x$.

(c) [3 points] Fill in the bottom row of the table. (Show your work here.)

Using the formulas in part (b) we have:

$$f(8) = \frac{1}{4}8^2 = 16$$
$$g(8) = -4(8) + 16 = -16$$
$$h(8) = 16\left(\frac{1}{2}\right)^8 = \frac{1}{16} = 0.0625.$$

7. The graph of a function f is shown below.



(a) [8 points] Fill in the table below with the indicated value or limit. If the value is undefined or the limit does not exist, place an X in the box.

a	$\lim_{x\to a^+} f(x)$	$\lim_{x\to a^-} f(x)$	$\lim_{x\to a} f(x)$	$f(a)$
-1	∞	-1	X	X
1	1	1	1	2

(b) [4 points] At what points in the domain $-3 < x < 3$, if any, is f not continuous, and why?

f is not continuous at $x = -1$ because f is not defined at $x = -1$ (also, $\lim_{x\to -1} f(x)$ does not exist).

f is not continuous at $x = 1$ because $\lim_{x\to 1} f(x) \neq f(1)$.

(c) [3 points] Is f an invertible function? Explain.

f is not invertible because it does not pass the horizontal line test. For example, the line $y = 1.5$ intersects the graph of f in two points.