Math 243: Mathematical Structures, Spring 2020 **Prof. Ballantine**

Homework Assignment # 10
DUE: Thursday, April 30, at 5:00pm in Moodle.

The numbered exercises refer to the manuscript *Mathematical Structures.* Always justify all assertions.

1. Exercise 9.3

2. Exercise 9.4 (You might need to show first that for every $\varepsilon > 0$ there exists an index N such that if $k \geq N$ then $|a_k - b_k| < \varepsilon$.)

3. Let $(a_k)_{k=2}^{\infty}$ be the sequence defined by $a_k = \dfrac{2k+1}{k-1}$. Use the definition of convergence to prove that (a_k) converges to 2.

4. Let $(a_k)_{k\geq 1}$ be a real sequence and a_∞ a real number. Consider the following conditions:

   (i) For every $\varepsilon > 0$, there exists an N such that if $k \geq N$ then $|a_k - \ell| < \varepsilon$.

   (ii) There exists an N such that for every $\varepsilon > 0$, if $k \geq N$ then $|a_k - \ell| < \varepsilon$.

   Are these conditions logically equivalent? If so, give a proof. If not, give an example of a sequence (a_k) and a real number a_∞ such that (i) holds but (ii) does not.