

Homework Assignment # 4

DUE: Thursday, February 20, at the **beginning** of classThe numbered exercises refer to the manuscript *Mathematical Structures*. Always justify all assertions.

1. Prove or disprove:

(a) $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$

(b) $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$

2. Exercise 2.12.

3. Prove that for all positive integers n , we have $1^2 + 3^2 + \cdots + (2n - 1)^2 = \frac{n(2n - 1)(2n + 1)}{3}$.4. Let $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$. Form the Cartesian product of A and B (give explicitly the elements of $A \times B$).5. For each of the following mappings, write out the image, $f(S)$, of S under f and the preimage, $f^*(T)$, of T under f for the given S and T , where $f : \mathbb{Z} \rightarrow \mathbb{Z}$.

(a) $f(x) = \begin{cases} x + 1 & \text{if } x \text{ is even} \\ x & \text{if } x \text{ is odd;} \end{cases} \quad S = \{0, 1, 5, 9\}, \quad T = 2\mathbb{Z} + 1 \text{ (the set of odd numbers).}$

(b) $f(x) = x^2; \quad S = \{-2, -1, 0, 1, 2\}, \quad T = \{2, 7, 11\}.$

(c) $f(x) = |x| - x; \quad S = T = \{-2, -1, 0, 2, 4\}.$