Homework Assignment # 5
DUE: Thursday, March 12, at the **beginning** of class

The numbered exercises refer to the manuscript *Mathematical Structures.* Always justify all assertions.

1. Use mathematical induction to prove that 3 divides $4^n - 1$ for all positive integers n.

2. For each mapping $f : \mathbb{Z} \to \mathbb{Z}$, determine whether the mapping is injective and/or surjective. Justify all answers.

   (a) $f(x) = x + 3$

   (b) $f(x) = 3x$.

   (c) $f(x) = x - |x|$

   (d) In each case, how does your answer change if the domain and the co-doamain are $\mathbb{R}$ (i.e., $f : \mathbb{R} \to \mathbb{R}$)? You only need to justify the answers that changed.

3. Use mathematical induction to prove that $1 + 2n \leq 3n$ for all positive integers n.

4. Let $f : \mathbb{Z} \to \mathbb{N}$ be defined by

$$f(n) = \begin{cases} 2n & \text{if } n \geq 0 \\ -2n - 1 & \text{if } n < 0. \end{cases}$$

   (a) Show that f is bijective.

   (b) Find the inverse mapping $f^{-1} : \mathbb{N} \to \mathbb{Z}$ of f.

5. Exercise 3.5