

## Homework Assignment # 6

DUE: Thursday, March 26, at at 5:00pm. Submit in Moodle.

The numbered exercises refer to the manuscript *Mathematical Structures*. Always justify all assertions.

1. Exercise 3.1.
2. Exercise 3.9.
3. Exercise 3.29 parts (a), (b), and (c).
4. Let  $n \in \mathbb{Z}^+$  and let  $G = n\mathbb{Z}$  be the set of multiples of  $n$ .
  - (a) Is addition a binary operation on  $G$ ? (I.e., is the sum of any two elements in  $G$  also in  $G$ ?)
  - (b) Is  $(G, +)$  a group? If so, is it Abelian?

**Note:** When you check conditions  $G2$  and  $G3$  from the definition of a group you have to make sure the identity you found is an element of  $G$  and for every  $a \in G$ , the inverse you found is an element of  $G$ .
5. Consider the binary operation  $*$  on  $\mathbb{Z}$  given by  $n*m = n+m+1$ . Prove that  $(\mathbb{Z}, *)$  is a group. What is the identity element? For an element  $a \in \mathbb{Z}$  what is its inverse? Is the group Abelian?
6. Let  $A$  be nonempty set.
  - (a) Does the binary operation  $\cup$  on  $\mathcal{P}(A)$  have an identity element? If so, what is it?
  - (b) Does the binary operation  $\cap$  on  $\mathcal{P}(A)$  have an identity element? If so, what is it?