Math 241

Quiz 3 Sample

February 26, 2008

Show any calculations that you do. Partial credit will be given.

1. Let $\mathbf{F}$ be the vector field $\mathbf{F}(x,y) = (2x - 5y, x - 2y)$ and let $\boldsymbol{\alpha}$ be the parametrically defined curve $\boldsymbol{\alpha}(t) = (\cos(t) + 2\sin(t), \sin(t))$. Show that $\boldsymbol{\alpha}(t)$ is a flow line of $\mathbf{F}(x,y)$.

2. Let $f(x,y) = xe^{xy}$.

   (a) Compute $\frac{\partial f}{\partial x}(x,y)$ and $\frac{\partial f}{\partial y}(x,y)$.

   (b) Let $\mathbf{u} = (u_1, u_2)$ be unit vector. Use your answer to (a) to compute $D_{\mathbf{u}}(f)(1,0)$.

3. Below is the contour plot of a function $z = f(x,y)$ on the domain $-3 \leq x \leq 3$ and $-3 \leq y \leq 2$. The contour curves are separated by one unit with values as labeled. The plot also contains an ellipse. Suppose that the ellipse is traversed *counterclockwise* starting at $(0, 1.2)$. As the ellipse is traversed, the values of $f(x,y)$ change.

(a) On the plot label those segments of the ellipse where $f(x,y)$ is increasing and those where $f(x,y)$ is decreasing as the ellipse is traversed counterclockwise.

(b) On the plot, label those segments of the ellipse where $f(x,y)$ is increasing most rapidly and those segments where $f(x,y)$ is decreasing most rapidly as the ellipse is traversed counterclockwise.

(c) On the plot mark and label those points on the ellipse where $f(x,y)$ has a local maximum along the ellipse and those points where $f(x,y)$ has a local minimum along the ellipse.

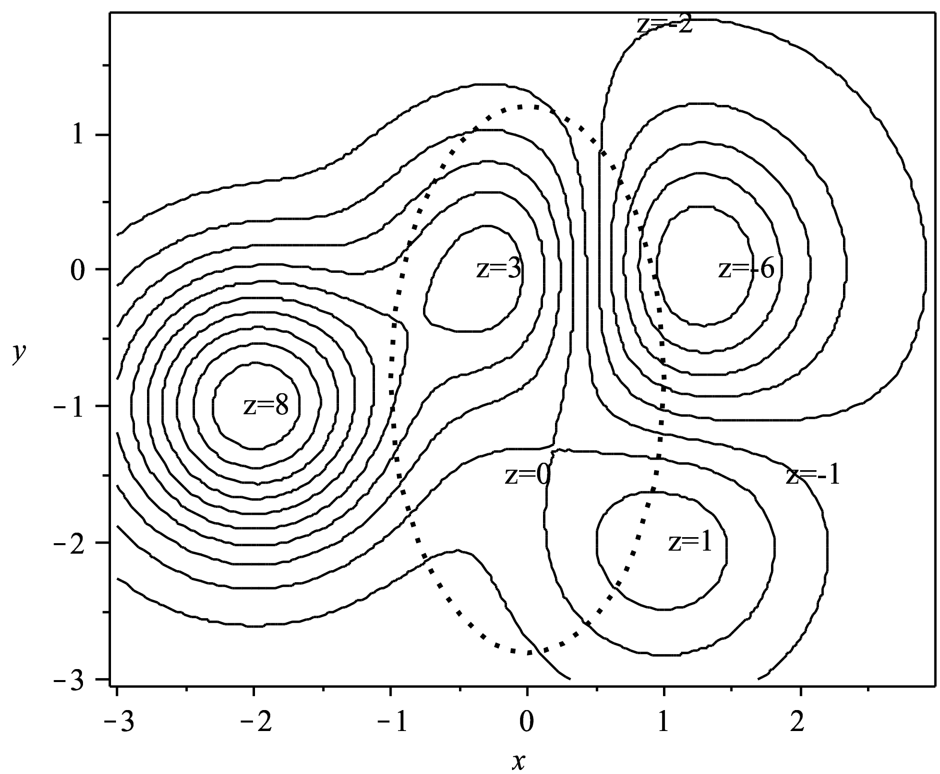

Figure 1: