MATH 363 – Topics in Topology

Spring 2007

MWF, 9:00 AM to 9:50 AM, Swords 330

Syllabus (1/15/07)

Instructor: Prof. David Damiano, 341 Swords, 793-2476/3374
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Office Hours: MT 1-2 PM, TTh 11 AM - Noon, Th 2-3 PM and by appointment.
Course Home Page: `http://math.holycross.edu/~dbd/math363/math363.html`

Course Materials (1) *Knots and Links*, Peter Cromwell. Publisher: Cambridge University Press, 2004. (2) *KnotPlot*. This is free software that you should download to your personal computer. It can be found at: `http://www.knotplot.com/`.

Prerequisites: MATH 242 and Math 244 or permission of the instructor.

Intended Audience: This course is designed for upper division mathematics majors or non-majors with knowledge of mathematics through the department's intermediate level courses. This course fulfills the department's Geometry/Topology breadth area requirement.

Introduction to Topology: Topology is a large area of mathematics that has interesting and important connections to geometry, analysis and algebra. Depending on one's perspective, the study of topology can take on the flavor of one of these other areas. As a reference point, geometry is the study of curves and surfaces in space and their analogues in higher dimensions. Two such objects are considered equivalent if one can be transformed into the other in such a way that distances between points on the object are preserved. That is, we think of objects as being rigid. By comparison, in topology, two objects are considered equivalent if one can be continuously deformed into the other. Consequently, we think of objects as being flexible rather than rigid. For example, from the point of view of geometry, an ellipse and a circle are different objects, but they are the same from the point of view of topology. Surprisingly, there is a great deal of structure embedded in this floppy definition of equivalence.

The main focus of study in this course will be closed curves in space. Since closed curves are equivalent to circles, they are not intrinsically interesting. However, the way a closed curve sits in space is very interesting. A curve in space can turn back on itself in a flexible manner so that it forms a *knot*. If there are two or more interwoven curves the result is called a *link*. The important questions involve how the curves are positioned in space up to topological equivalence. Our study of knots and links will begin with simple, visual approaches to this question and then move on to more sophisticated algebraic approaches. Since knot theory has been the object of study for over 100 years, many of these ideas can be considered classical. On the other hand, there have been many important developments in knot theory in the past 20 years, including applications to molecular biology and chemistry.

In the first two weeks of the semester, we will develop basic topological notions not discussed in the text. These include topological space, continuity, homeomorphism, connectedness, path connectedness, and compactness. We will spend the next two weeks on the examples and properties of topological spaces presented in Chapter 2 of the text. The remainder of the course will be devoted to the study knots and links, which is the main focus of the text. We will go through the chapters in the following order: 1, 3, 4, 7.4-7.11, 8, 9, 10, 5, 6, and 7.1-7.3. Chapters 1, 3, and 4

provide an introduction to knots and links. The remaining chapters present the material in a rough chronological order. By rearranging the chapters, we will be able to spend more time on recent developments in the subject. Chapters 5, 6 and 7.1-7.3 contain important connections between knots, surfaces, linear algebra and graph theory. Although this material comes earlier in the text, the constructions are more involved than those that come later. In addition, the material in later chapters provides more topics for projects, so that it will be helpful to cover it earlier in the semester.

Class Format: Most of the classes will be devoted to lecture. However, because the class is small, it will be possible for you to present homework at the blackboard on a regular basis. In addition, at the end of the semester, there will be opportunities for the presentation of projects.

Homework: There will be approximately ten homework assignments from the text or handouts. A schedule of assignments appears at the end of the syllabus.

Project: There will be a semester project that can be done individually or in teams. The tentative schedule for the projects is as follows: Topics chosen by Wednesday, March 28, March 16, project outline due Monday, March 26, first draft due Monday April 23, project due Monday, April 30 oral reports Wednesday, April 25 to Monday, April 30 in class.

Exams: There will be one in-class exams that focus on definitions and statements of theorems. This will contain some short answer questions to test your understanding, but will not involve extended proofs. This is tentatively scheduled for *Friday, February 23.* (With the agreement of the class, this may be scheduled in the evening.) The final exam will be a 2-hour exam during the regularly scheduled exam period and will have the same format. The final exam time slot for this time period is *Monday, May 7, 8:30 AM - 11:30 AM.* (The time period may be moved by the Registrar to avoid construction noise.)

Grading: There are several components to the course grade.

Homework	50%
Project	20%
Mid-term Exam	10%
Final Exam	20%
Total	100%

Academic Honesty: The Department of Mathematics and Computer Science adheres to the College's policy on Academic Honesty, which may be found on page 13 of the 2006-2007 College Catalogue. In addition, the department has formulated the attached statement intended to amplify the policy as to how it might apply in mathematics and computer science.

Semester Schedule (as of 1/15/07): Any changes will be announced in class well before due dates.

- Friday, January 26: Assignment 1 due.
- Friday, February 2: Assignment 2 due.
- Friday, February 9: Assignment 3 due.
- Friday, February 16: Assignment 4 due.
- Friday, February 23: **Hour Exam**.
- Friday, March 2: Assigment 5 due.
- Friday, March 16: Project topics selected.
- Friday, March 23: Assignment 6 due.
- Wednesday, March 28: Project outline due.
- Wednesday, April 4: Assignment 7 due.
- Friday, April 20: Assignment 8 due.
- Monday, April 23: First draft of project report due.
- Wednesday, April 25–Monday, April 30: Oral presentations of projects in class.
- Monday, April 30: Project report due.
- Monday, May 7: **Final exam** 8:30 AM – 10:30 AM.