# MATH 363 – Topics in Topology

Spring 2020
MWF Noon to 1 PM, Swords 328

## Syllabus (1/21/20)

**Instructor:** Prof. David Damiano, 341 Swords, 793-2476/3374
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**Office Hours:** Monday 11 AM - Noon, Tuesday 1-2 PM, Wednesday 10 AM - Noon, Thursday 1 - 3 PM, and by appointment.

**Course Home Page:** `http://math.holycross.edu/~dbd/math363/math363.html`

**Course Materials:** Textbook: *Essential Topology*, by Martin D. Crossley.
Publisher: Springer; 2010. Corrected Printing
ISBN (Paperback): 978-1-85233-782-7 (Paperback)
(Available on-line through Amazon.com. Used copies will be fine.)

**Prerequisites:** MATH 241 and Math 244 or permission of the instructor.

**Intended Audience:** This course is designed for upper division mathematics majors or non-majors with knowledge of mathematics through the department's intermediate level courses. This course is an elective 300 level course and carriesthe Project Course designation.

**Introduction to Topology:** Topology is a large area of mathematics that has interesting and important connections to all areas of mathematics and many sciences. As a reference point, geometry is the study of curves and surfaces in space and their analogs in higher dimensions. Two such objects are considered equivalent if one can be transformed into the other in such a way that distances between points on the object are preserved. That is, we think of objects as being rigid. For example, geometrically a circle and ellipse are not equivalent. By comparison, in topology, two objects are considered equivalent if one can be continuously deformed into the other. Consequently, we think of objects as being flexible rather than rigid. In topology, a circle and ellipse are equivalent. There are similar examples involving higher dimensional objects–topology can't tell the difference between a football and a basketball, but it "knows" the two are hollow.

Historically topology has been thought of as one of the more theoretical subjects within mathematics and applications have been to other areas of mathematics. The lone exception to this is applications to theoretical physics, which is the most mathematical area within physics. However, in the past 20 years, topological methods have been used in a variety of applications involving data and networks that have been made possible by the development of efficient algorithms. This area of topology is called Topological Data Analysis (TDA) or Computational Topology (CT). The goal of this course will be to cover the standard material needed to develop and apply the basic ideas of TDA. The group projects at the end of the course will all involve TDA. This will necessarily involve learning to use MATLAB and TDA software.

**Topics Covered:** The text, *Essential Topology*, follows a standard order of topics and builds on concepts in analysis, though no prior knowledge of analysis is required. We will cover roughly half the material in the book, though we switch the order in order to cover sections necessary for the projects early in the course. Here is the order of topics including material from TDA that is not in the text (denoted by NFT):

- Metric spaces (NFT)
- Convex sets (NFT)
- Simplicial and Cubical complexes (Sections 7.1, 7.2 and material NFT)
- Simplicial Homology (Chapter 9)
- Persistent Homology (NFT)
- Representations of Persistent Homology (NFT)
- Applications of Persistent Homology (NFT)
- Point-Set Topology Basics (Chapters 2.3 through 4)
- Constructions of Topological Spaces (Chapter 5)
- Homotopy Equivalence (Sections 6.1 and 6.2)

**Class Format:** In addition to lectures, there will be regular collaborative activities in class. Some of these will form the basis of homework assignments. We will also hold several class meetings in the Starting after the hour exam we will meet in Swords 219 on occasion to learn how to use Matlab, a widely used mathematical software designed for sophisticated analysis of data, and JavaPlex, a computational topology software package callable from Matlab. Computer assignments will be completed in teams.

**Homework:** There will be weekly homework and/or group assignments, mostly due on Fridays. Some of the assignments will involve use of Matlab and JavaPlex. You will have at least a week to complete each assignment.

**Proof Portfolio:** This is a collection of 10 to 12 proofs that you will have to submit at the end of the semester. It will consist of homework solutions covering that number of different topics. The purpose is to demonstrate mastery of various proof techniques. The topics will be assigned as we go along and will be part of ordinary homework. There will first be a peer review in class with a partner. Based on this review, you will be able to rewrite your proof for submission to me. I will provide comments for one further rewrite, which you will submit in your portfolio at the end of the semester. (So there will be three versions of each proof.) The Proof Portfolio will be due Friday, May 1.

**Colloquia Reports:** There will be three job candidate talks, three colloquiua talks, the Sulski Lecture, and the year-end department banquet lecture, for a total of 8 lectures during the semester. I would like you to attend five of these: Two of the three job candidate talks, at least one colloquium, and and at least one of the Sulski Lecture and banquet lecture. The fifth talk can be of your choosing. This spreads the talks over the semester. If you're unable to attend these lectures because of a class, work, or practice conflict, I can suggest videos that you can watch at your leisure. In order to get credit for attending a talk, you should send me a one or two paragraph response commenting on the mathematical content of the talk.

**Hour Exam**: There will be one exam tentatively scheduled for the evening of **Wednesday, March 18** at a time to TBA. This will be on material that we cover up to and including Friday, March 13. It will focus on definitions, statements of theorems, and short proofs. There is NO final exam for the course.

**The Project:** In lieu of a final exam there will be a final group project in teams of three students. I will provide a short list of suggested topics. Here is a tentative schedule of project deadlines. After spring break we will set firm dates:

- 3/16 Monday: Choose project teams
- 3/23 Monday & 3/24 Tuesday: Meet as a group in office hours to discuss possible project topics
- 4/1 Wednesday: Submit project topics along with a half page description or abstract
- 4/13 Monday: Project updates in office hours
- 4/15 Wednesday: 5 minute in-class report with 4-6 slides PowerPoint or Beamer
- 4/27 Monday: Project updates in office hours
- 4/27 Monday: Presentations submitted for review.
- 5/1 Friday: Draft final report submitted.
- 5/1 Friday, 5/4 Monday, and 5/5 Tuesday: In-class project presentations (25-30 min) using PowerPoint or Beamer.
- 5/8 Friday: Project reports due.

**Grading:** There are several components to the course grade.

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| Homework | 25% |
| Collaborative Assignments | 10% |
| Software Assignments | 10% |
| Proof Portfolio | 5% |
| Colloquia Reports | 5% |
| Midterm Exam | 20% |
| Final Project | 25% |
| Total | 100% |

**Academic Honesty:** The Department of Mathematics and Computer Science adheres to the College's policy on Academic Honesty, which may be found in the College Catalogue. In addition, the department has formulated the attached statement intended to amplify the policy as to how it might apply in mathematics and computer science.