Probability: Review Sheet

Math Properties to Remember

- Multiply fractions straight across: $\frac{2}{3} \cdot \frac{4}{2} = \frac{2 \cdot 4}{3 \cdot 2} = \frac{8}{6}$
- Multiplying numbers with exponents
    - If the base is the same, add the exponents: $3^4 \cdot 3^5 = 3^9$
    - If the base isn't the same, you can only simplify with algebra: $2^2 \cdot 3^2 = 4 \cdot 9 = 36$
- Thus, $\left(\frac{3}{5}\right)^3 \cdot \left(\frac{3}{5}\right)^7 = \left(\frac{3}{5}\right)^{10}$

Definitions

- Two things are considered **independent** if the chances for the second given the first are the same, no matter how the first one turns out. (Otherwise, the two events are *dependant.*)
    - Two tosses of a coin are *independent*. The outcome of the second toss is equally likely heads or tails, no matter the outcome of the first toss.
    - If two events are independent, the chance that **both** will occur is the **product** of their chances. This is the **multiplication rule for independent events**.

- Outcomes are considered **mutually exclusive** when the occurrence of one prevents the occurrence of another; one excludes the other.
    - A card drawn from the top of a deck could be a diamond, heart, spade, or club. These four possibilities are *mutually exclusive* because if one occurs, the others cannot. If the card is a club, it cannot also be a diamond, spade, or heart.
    - If two events are mutually exclusive, the chance that one **or** the other will occur is the **sum** of their chances. This is the **addition rule for mutually exclusive events**.

- The chance that two things will both happen equals the chance that the first will happen multiplied by the chance that the second will happen given that the first happened. The first chance is an **unconditional chance** and the second is a **conditional chance**.
    - Two cards are drawn from the top of a deck. The chance that the first card will be a King of Hearts and the second card will be the Queen of Hearts is the chance of the first, $\frac{1}{52}$, multiplied by the chance of the second, $\frac{1}{51}$.
    - This is the **product rule for conditional chance**.

- **Factorial (!)** is a mathematical function that tells you to multiply together all of the integers from 1 up to that number:

  $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

  So, in general,

  $N! = N \cdot (N-1) \cdot (N-2) \cdot \ldots \cdot 1$

  Note: 0! is defined as 1.

  Some properties to consider:

  $(4!) \cdot 5 = 5!$ and $(4!) \cdot 5 \cdot 6 = 6!$ (using division, we can simplify) $\frac{4! \cdot 5 \cdot 6}{4!} = \frac{6!}{4!}$ so $5 \cdot 6 = \frac{6!}{4!}$

- **Binomial Coefficient**, also called "n choose k," is a mathematical formula $\frac{n!}{k!\,(n-k)!}$.
  - The symbol $\binom{n}{k}$ is used to represent this fraction. Caution: This symbol is not the same as $\left(\frac{n}{k}\right)$

  Here is the calculation for "7 choose 3":

  $$= \binom{7}{3} \; \frac{7!}{3!\,(7-3)!} = \frac{7!}{3! \cdot 4!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(3 \cdot 2 \cdot 1)(4 \cdot 3 \cdot 2 \cdot 1)} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = \frac{210}{6} = 35$$

How to use these functions

- Factorials are used to calculate the ways things can be arranged in order. For example, the number of ways to arrange 7 children in a line is

  $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$ ways.

- The binomial coefficient is used to calculate ways to arrange a portion of a whole. For example, the number of ways to arrange 3 quarters and two dimes in a row, not paying attention to which quarter goes in a "quarter spot" and which dime goes in a "dime spot" is

  $$\left(\frac{5}{3}\right) = \frac{5!}{3!\,(5-3)!} = \frac{5!}{3! \cdot 2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(3 \cdot 2 \cdot 1)(2 \cdot 1)} = \frac{5 \cdot 4}{2 \cdot 1} = \frac{20}{2} = 10$$

An example

The binomial coefficient can also be used to calculate the chance of an event occurring a certain number of times in a sequence of events. For example, the chance that a die is tossed five times and exactly three of the tosses turn out to be 4:

"5 choose 3" multiplied by $\frac{1}{6}$ three times and $\frac{5}{6}$ two times. The $\frac{1}{6}$ is the chance that a single die comes up 4 and $\frac{5}{6}$ is the chance that the singe die **does not** come up 4.

This is $\binom{5}{3} \cdot \left(\frac{1}{6}\right)^3 \cdot \left(\frac{5}{6}\right)^2 = \frac{5!}{3!(5-3)!} \cdot \left(\frac{1}{6}\right)^3 \cdot \left(\frac{5}{6}\right)^2 = \frac{5!}{3! \cdot 2!} \cdot \frac{1^3 \cdot 5^2}{6^5} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(3 \cdot 2 \cdot 1)(2 \cdot 1)} \cdot \frac{5^2}{6^5}$
$= \frac{5^3 \cdot 4}{6^5 \cdot 2 \cdot 1} = \frac{500}{15552} = .0321$

Notice we used the calculation above to simplify the binomial coefficient and the rules for multiplying fractions to simplify the factors for chance.