MATH 131, Sections 02 and 04, Fall 2005

Computer Lab #2

A New Derivative?

DUE DATE: Friday, Oct. 7th, in class.

The goal for this lab project is to develop a better understanding of the derivative from a graphical, numerical and analytical viewpoint. We will use Maple to draw tangent lines and estimate the derivative. A possible new formulation of the derivative is presented and you will need to use both Maple **and** hand calculations to decide whether it represents the derivative or is abstract nonsense. Hopefully, you will find this last question interesting and informative.

It is **required** that you work in a group of two or three people. Any help you receive from a source other than your lab partner(s) should be acknowledged in your report. For example, a textbook, web site, another student, etc. should all be appropriately referenced. Please turn in one report per group, listing the names of the groups members at the top of your report. Be sure to answer all questions carefully and neatly, writing in complete sentences. You should turn in your answers on separate sheets of paper. There are four plots via Maple which need to be turned in for this project.

1. Recall that the derivative of a function at a point is the slope of the tangent line to the curve at that point. Use Maple to define the function $f(x) = x^{\sin x}$ and then plot it over the domain $0 \leq x \leq 2$. The commands for this are listed below if you have forgotten them. Be sure to type them in exactly as below.

```
f := x -> x^(sin(x));
plot(f, 0..2);
```

Next, use Maple to define the function (where have you seen this expression before?)

$$g(h) = \frac{f(1+h) - f(1)}{h}$$

NOTE: You can evaluate this or any function by typing `g(0.2);` for example, to obtain the function value at $h = 0.2$. Use these two functions to answer the questions below.

(a) By changing the domain of your plot range, zoom in on the graph of f to estimate the value of $f'(1)$. Print out and turn in the graph you use to estimate $f'(1)$.

(b) Using Maple, evaluate $g(h)$ at the h-values $-0.01, -0.001, -0.0001, 0.0001, 0.001, 0.01$ (make a table). Based on your calculations, estimate

$$\lim_{h \to 0} g(h).$$

(c) Compare your answers from part (a) and part (b). What do you notice? Explain why.

(d) Using your value for $f'(1)$ from part (b), compute the equation of the tangent line to $f(x)$ at the point $x = 1$. (You do not need Maple to do this!)

(e) Draw a plot containing both the graph of $f(x)$ and the tangent line from part (d) over the interval $0.5 \leq x \leq 1.5$. Recall that to plot two functions (say f and l) on the same set of axes, you type `plot({f,l},0.5..1.5);` Turn in a print out of your plot. What happens to your graph as you zoom in around $x = 1$?

2. Repeat the above questions (parts a through e) for the function $f(x) = x^x$. You should keep the same function g as above, but this time it will be defined in terms of your new function f. You will need to execute the new f and then execute g. Otherwise Maple will think that you are still using the old f.

3. We will now investigate an alternative formulation of the derivative suggested by a former Calculus student. Consider the following limit expression

$$\lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h} \tag{1}$$

Although it seems reminiscent of the formula for the derivative of $f(x)$, it is not the same expression. Your job is to determine whether this limit formula is an alternative expression for the derivative or is just some useless mathematical nonsense. The first THREE questions below are conceptual and are to be done **without** Maple.

(a) Find the slope of the line between the points $(x-h, f(x-h))$ and $(x+h, f(x+h))$. Relate your answer to the limit expression (1) above.

(b) Draw a graph of an arbitrary function f (you make it up) and plot the points $(x-h, f(x-h))$ and $(x+h, f(x+h))$ where h is taken to be some small positive number (x is arbitrary). Your class notes on the derivative may be helpful here. Draw the line through the two points. What happens to this line as h approaches 0?

(c) Suppose that we define $f(x) = x^2$. Using some algebra, evaluate the above limit (1) with this particular function f. Compare your answer with $f'(x)$. What do you notice?

(d) Using Maple and the function $f(x) = x^{\sin x}$ from Problem 1, define the function

$$g(h) = \frac{f(1+h) - f(1-h)}{2h}$$

As in Problem 1 part (b), use Maple to evaluate $g(h)$ at the h-values $-0.01, -0.001, -0.0001, 0.0001, 0.001, 0.01$ (make a table). Compare your answers to those you found from Problem 1 part (b). Estimate

$$\lim_{h \to 0} g(h).$$

(e) Using Maple and the function $f(x) = x^x$ from Problem 2, define the function

$$g(h) = \frac{f(1+h) - f(1-h)}{2h}$$

As in Problem 2 part (b), use Maple to evaluate $g(h)$ at the h-values $-0.01, -0.001, -0.0001, 0.0001, 0.001, 0.01$ (make a table). Compare your answers to those you found from Problem 2 part (b). Estimate

$$\lim_{h \to 0} g(h).$$

(f) (THE PUNCHLINE.) Based on your answers for parts (a) through (e), make a conjecture as to the effectiveness of using the limit expression (1) as a formula for the derivative of a function. Is the formula useful or abstract nonsense? Explain in detail. If you decide on useful, does it provide a better or worse method of approximating the derivative? Explain.