MATH 136-04, Fall 2010

Computer Lab #1

Lissajous Figures

DUE DATE: Monday, September 20, Start of class

The goal for this lab project is to further your understanding of parametric equations by exploring a special type of curve called a **Lissajous figure**. Using the computer software package Maple, you will draw and animate several different figures by varying certain **parameters** in the defining equations. Based on your findings, you will make some predictions and draw some conclusions about the effect these parameters have on the corresponding Lissajous figure. Through this exploration you should develop a better understanding of parametric curves, in particular, the connection between the two component functions $x(t)$ and $y(t)$ and the path traced out by the particle in the xy-plane.

It is **required** that you work in a group of two or three people. Any help you receive from a source other than your lab partner(s) should be acknowledged in your report. For example, a textbook, website, another student, etc. should all be appropriately referenced. Please turn in **one report per group**, listing the names of the groups members at the top of your report. Be sure to answer all questions carefully and neatly, writing in complete sentences. Your report should be TYPED and you are encouraged to type all of it in your Maple worksheet. There are **ONLY TWO** Maple plots which need to be turned in for this project. You are welcome to turn in a few more but please don't overload your report with extra, unasked for graphs!

Lissajous Figures

The French physicist Jules Antoine Lissajous (1822-1880) (pronounced Lee-suh-zhoo) was interested in studying waves and vibrations. Attaching mirrors to two tuning forks which were vibrating at *different* frequencies, he was able to project light off of the mirrors and onto one screen to create what is today called a **Lissajous figure**. His setup was similar to the modern device used to project laser light shows. Lissajous figures were used in the old days to determine the frequencies of sounds or radio signals. They found their way into popular culture in many sci-fi movies and TV shows, including the opening footage for *The Outer Limits* TV series. ("Do not attempt to adjust your picture — *we* are controlling the transmission.") [1,2].

To create Lissajous figures, we will use the parameterization

$$\begin{aligned} x &= \cos\left(\frac{2\pi}{a}\,t\right) \\ y &= \sin\left(\frac{2\pi}{b}\,t\right) \end{aligned}$$

where a, b are always taken to be positive real numbers. The equations are chosen so that the period of the horizontal component x is a and the period of the vertical component y is b. To begin, define the parameterization by typing

```
r := t -> (cos(2*Pi*t/a),sin(2*Pi*t/b));
```

exactly into Maple. It should appear in a mathematical format. You may click on the "cos" and "sin" items in the **Expression Palette** instead of typing them by hand. Each time you draw a Lissajous figure, you will save a lot of time if you redefine a and b as you go. Be sure to hit return after you change their values, otherwise Maple will continue to use the previous values. Assuming you have defined r correctly, the commands

```
a := 3: b := 1:
plot([r(t),t=0..2]);
```

should draw the Lissajous figure with $a = 3$ and $b = 1$ over the time interval $0 \leq t \leq 2$. Try plotting a few sample figures with different a and b values over different time intervals. What happens as you increase the time interval?

In addition to plotting a parametric equation, it is possible to animate a parametric curve using the Maple command `animatecurve`. For example, executing

```
with(plots):
animatecurve([r(t),t=0..2],frames=30,numpoints=75);
```

will provide a set of coordinate axis with no graph. However, clicking on the graph with the mouse will then give a DVD style play/pause control panel at the top of the screen. You can use this to animate the curve. Try pressing play to see the curve animated. Note that you only need to load the package `plots` once in order to use the `animatecurve` command. The `frames=30` command gives the number of frames viewed throughout the total animation. Increasing this number will give more frames to your "movie" but it will take longer to watch the entire process unfold. The `numpoints=75` command controls the number of points plotted when producing the curve. Higher numbers yield more accurate plots but take longer to produce.

Suggestion: When you want to produce a "nice" plot of a particular Lissajous Figure, simply use the `plot` command. On the other hand, if you want to watch the curve being formed as time progresses, use the `animatecurve` command.

1. Begin by setting $a = b = 1$. What figure do you obtain? Using the parameterization above, what is the **period** of this figure? By period, we mean how long does it take until the trajectory returns to its starting position and direction. What figure will you obtain whenever $a = b$? Explain. No graphs need to be turned in for this question.

2. Use Maple to plot the Lissajous figure for $a = 2$ and $b = 1$. Print out your plot and label at least 5 points on the figure with their corresponding times. What is the period of this curve?

3. Keeping b fixed at 1 while varying a to $3, 4, 5, \ldots$, use Maple to plot the corresponding Lissajous curves. What changes do you see? Can you explain why they occur? What is the period when $b = 1$ and $a = 3$? What is the period when $b = 1$ and $a = k$, for any positive integer k?

4. Use Maple to plot the Lissajous figure for $a = 1$ and $b = 2$. How is this different from the plot in Question 2? Explain. Would you say this curve is periodic? In other words, does it ever return to its starting point and then retrace its path again? What is the period of this curve? What shape is this curve? Find an equation between x and y which describes this curve? (*Hint:* Use a trig identity.) Print out your plot and label at least 5 points on the figure with their corresponding times.

5. Keeping a fixed at 1 while varying b to $3, 4, 5, \ldots$, use Maple to plot the corresponding Lissajous curves. What changes do you see? Can you explain why they occur? What is the period when $a = 1$ and $b = 3$? What is the period when $a = 1$ and $b = k$, for any positive integer k? Do

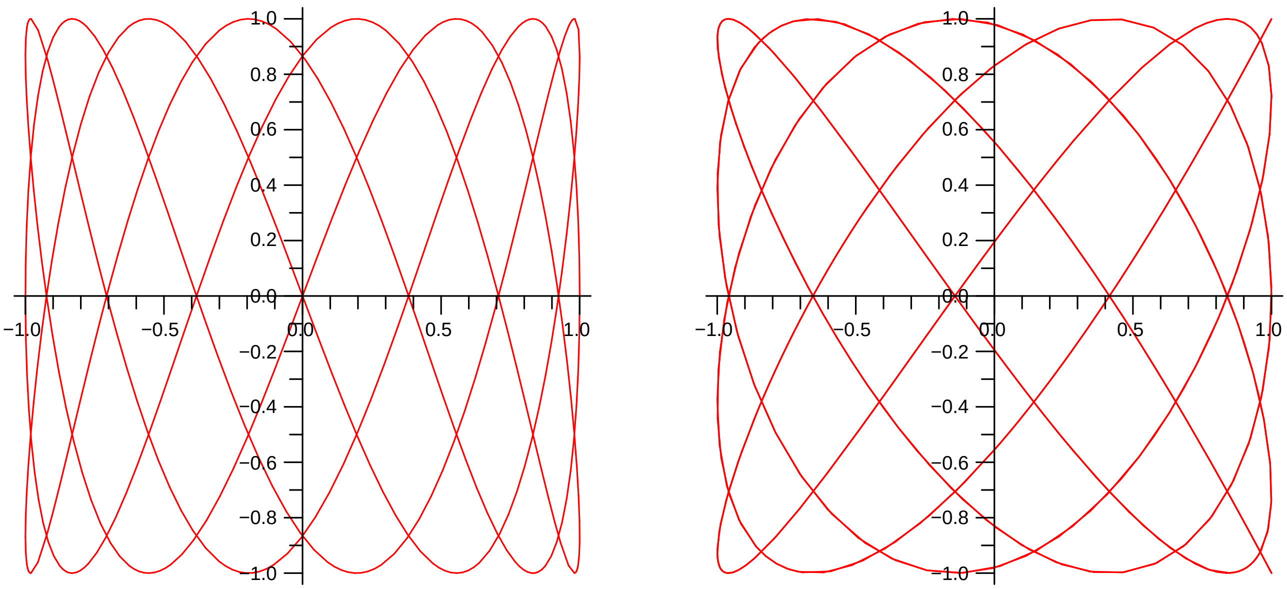

Figure 1: Two typical Lissajous figures. Can you find the particular integer values of a and b used in each case?

all the Lissajous curves obtained in this part close up on themselves (complete a cycle)? Which ones close up and which ones don't? Can you explain why?

6. By experimenting with different integer values of a and b other than the case $a = b$, what can you conclude about the corresponding Lissajous figure? What role do a and b play? Try fixing one of a or b constant and varying the other to look for patterns in the figures. (Much of mathematical research begins with looking for patterns!) What is the period of the Lissajous figure if $a = 5$ and $b = 3$? What is the period of the Lissajous figure if $a = 4$ and $b = 6$? Find an expression for the period given any two integer values for a and b.

7. The two Lissajous figures in Figure 1 were created using the $r(t)$ parameterization given above. Find the integer values of a and b used in each case. Explain how you obtained your answers. Note that the figure on the left closes up on itself while the figure on the right does not.

8. All the examples above have used a and b as integers. What happens if you try $a = 1$ and $b = \sqrt{2}$? The syntax for the square root is `sqrt(2)` or you can choose it from the expressions palette. Is the curve periodic? What happens as you plot the curve over longer and longer time intervals? Explain.

References

1. Hobbs Ed., *Lissajous Lab*, website: "http://www.math.com/students/wonders/lissajous/ lissajous.html" maintained by Math.com.

2. MacTutor History of Mathematics archive, *Jules Antoine Lissajous*, website: "http://www-groups.dcs.st-andrews.ac.uk/history/Mathematicians/Lissajous.html" maintained by the School of Mathematics and Statistics, University of St. Andrews, Scotland.