

Topics in Mathematics: Math and Music

Section 2.4: Key Signatures, and the Circle of Fifths

The Circle of Fifths

The *Circle of Fifths* is a special musical clock with C located at the top. Moving clockwise, each ensuing notch on the twelve-note clock is a perfect fifth above the previous note. This yields the sequence C, G, D, A, E, B, F \sharp (at the bottom of the circle) and C \sharp . In the opposite direction, moving counterclockwise around the circle means lowering the pitch by a perfect fifth. Beginning at the top of the clock, this gives the sequence F, B \flat , E \flat , A \flat , D \flat , G \flat (at the bottom of the circle) and C \flat (see Figure 1).

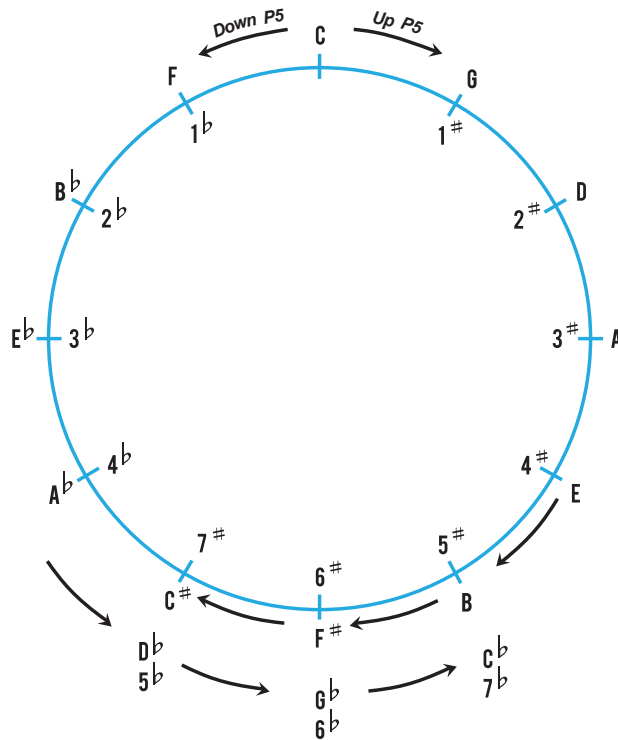


Figure 1: The circle of fifths. **Memorize this figure!**

Advantages and key aspects of the circle of fifths:

- (i) Useful device for learning the number of sharps or flats in a given major scale (or key).
- (ii) Tonal proximity: Keys near each other on the circle are also musically close, as they share several notes in common (see Figure 2).
- (iii) Recall that there are 7 half steps in a perfect fifth, and 12 in an octave. Because 7 and 12 are relatively prime, the circle of fifths contains all 12 notes of the chromatic scale (it does not close up too early). Going up 12 perfect fifths is equivalent to going up 7 octaves: $12 \cdot 7 = 7 \cdot 12 = 84$. There are 88 keys on the piano so it is possible to demonstrate this on the piano.

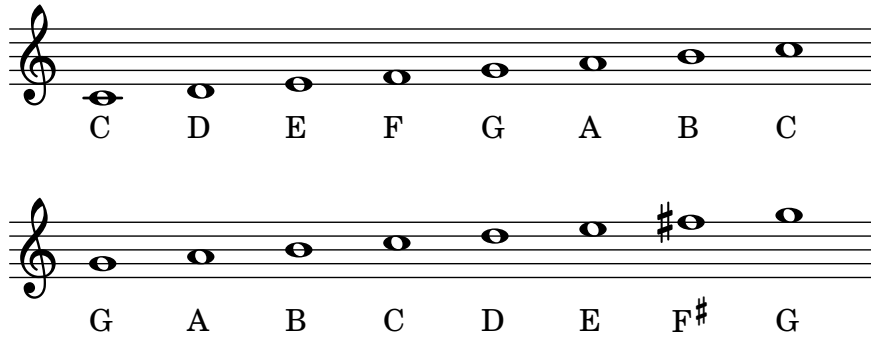


Figure 2: Tonal proximity: The C major and G major scales have the same set of notes except the F in the key of C becomes an F \sharp in the key of G.

(iv) Notice that when raising every note on the piano keyboard by a perfect fifth, all the white keys map to white keys, except B goes to F \sharp , and all the black keys map to other black keys, except B \flat is sent to F. This is why the circle of fifths works (see text for detailed explanation).

The Sharp Keys

The major scales containing only sharps are listed in Figure 3. As we go around the circle of fifths in the clockwise direction, we add a new sharp at each step. This sequence of sharps is important and is demonstrated by the third column in the figure.

<u>Major Scale</u>	<u># of Sharps</u>	<u>Sharp Sequence</u>
C	0	
G	1	F \sharp
D	2	F \sharp , C \sharp
A	3	F \sharp , C \sharp , G \sharp
E	4	F \sharp , C \sharp , G \sharp , D \sharp
B	5	F \sharp , C \sharp , G \sharp , D \sharp , A \sharp
F \sharp	6	F \sharp , C \sharp , G \sharp , D \sharp , A \sharp , E \sharp
C \sharp	7	F \sharp , C \sharp , G \sharp , D \sharp , A \sharp , E \sharp , B \sharp

Figure 3: The sharp keys and their sequence of sharps.

The Flat Keys

The major scales containing only flats are listed in Figure 4. This time, as we go around the circle of fifths in the counterclockwise direction, we add a new flat at each step. This sequence of flats is important and is demonstrated by the third column in the figure.

<u>Major Scale</u>	<u># of Flats</u>	<u>Flat Sequence</u>
F	1	B ^b
B ^b	2	B ^b , E ^b
E ^b	3	B ^b , E ^b , A ^b
A ^b	4	B ^b , E ^b , A ^b , D ^b
D ^b	5	B ^b , E ^b , A ^b , D ^b , G ^b
G ^b	6	B ^b , E ^b , A ^b , D ^b , G ^b , C ^b
C ^b	7	B ^b , E ^b , A ^b , D ^b , G ^b , C ^b , F ^b

Figure 4: The flat keys and their sequence of flats.

Key Signatures

Each major key has a unique number of flats or sharps. There is only one key with four flats and it is A^b major. Instead of writing the flats on the staff every time a flatted note appears, we use a *key signature* at the start of each line to indicate which notes should be flatted or sharped. The accidentals in a key signature (see Figure 5) are listed in the same order as shown in the third column of the previous two figures.

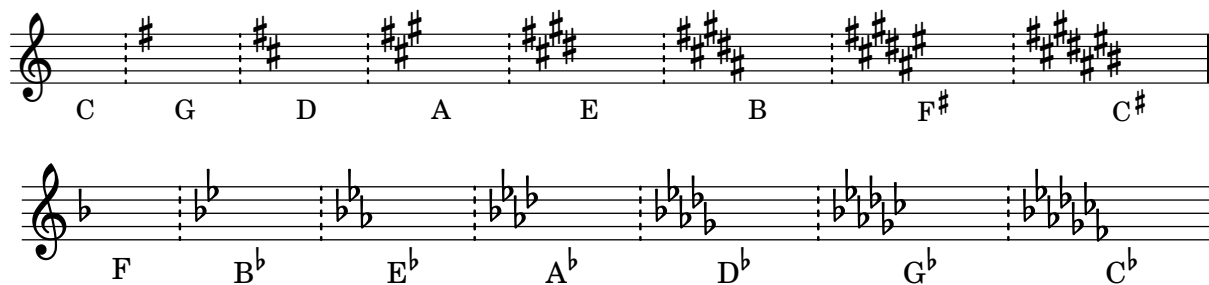


Figure 5: Key signatures for the sharp and flat keys.

Using key signatures helps simplify the musical notation. For example, the first four measures of *Twinkle Twinkle Little Star* are shown in the key of F \sharp major with and without a key signature (see Figure 6). Note how much simpler the music with a key signature is to read.

Important: The accidentals in a key signature apply to every note with the same name, regardless of which octave it resides in. For instance, if there is an F \sharp in the key signature, then *every* F on the staff (no matter where it is located) is sharped.



Figure 6: The first four bars of the lullaby *Twinkle Twinkle Little Star* in the key of F \sharp major, written with the key signature (bottom) and without (top).

Exercise 1: Figure 7 shows an excerpt of music using a key signature. Indicate which notes receive accidentals and give the names of those notes.



Figure 7: Indicate which notes receive accidentals according to the key signature and give the names of those notes.

The Relative Major of a Minor Key

There is an important connection between major and minor keys. For example, both the C minor scale and the E \flat major scale contain the same three flats (B \flat , E \flat , and A \flat). In fact, the total collection of notes is identical, just written in a different order (see Figure 8).

The *relative major* of a natural minor key has the same number of flats or sharps as its minor counterpart. The key of the relative major is found by going up a minor third from the tonic of the minor scale. Similarly, the *relative minor* of a major key is the minor key with the same number of flats or sharps as its major counterpart. The key of the relative minor is found by going down a minor third from the tonic of the major scale.

For example, the relative minor of C major is A minor, since A is a minor third *below* C. Both C major and A minor have no sharps or flats. Just as with the major keys, enharmonic equivalence allows for certain minor keys to share the same notes, but have different names. The relative major of E \flat minor is G \flat major because G \flat is a minor third above E \flat . Thus, the key of E \flat minor has six flats. On the other hand, the relative major of D \sharp minor is F \sharp major, with six sharps. Figure 9 connects the relative minor keys with their major counterparts in an “ancient” version of the circle of fifths.

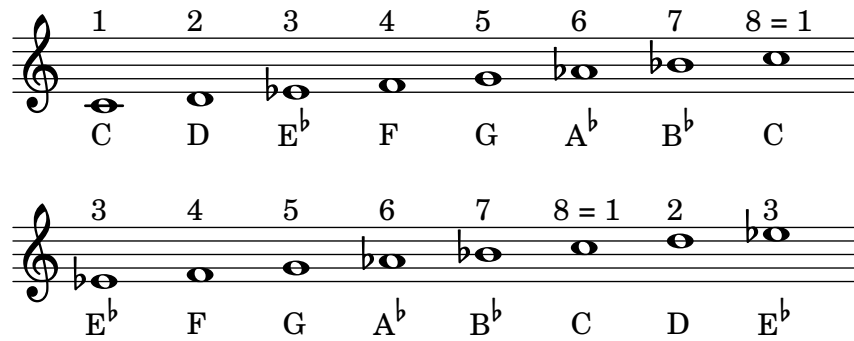
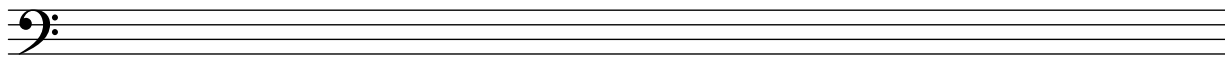
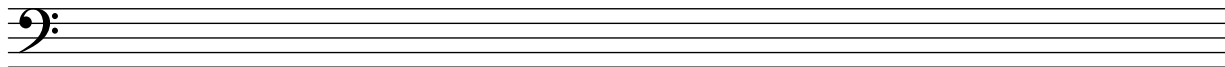


Figure 8: Taking the notes of the C minor scale and starting on the third scale degree yields an E^b major scale.

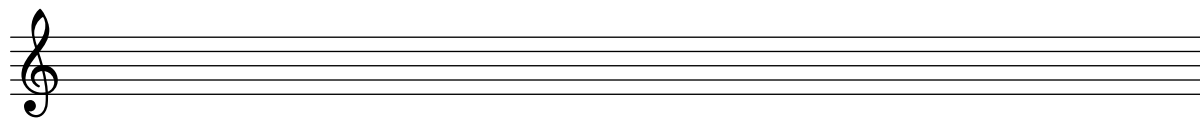
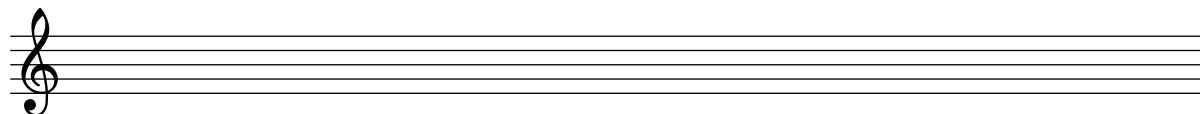


Figure 9: An “ancient” version of the circle of fifths indicating the relative major and minor keys. Image by Josh Wells, Odd Quartet Comics.

Exercise 2: What is the relative minor of D^b major? Write out a D^b major scale (ascending, bass clef) and its relative minor scale and check that each scale has the same overall set of notes (different ordering, but same collection of pitches).



Exercise 3: What is the relative major of F[#] minor? Write out an F[#] minor scale (ascending, treble clef) and its relative major scale and check that each scale has the same overall set of notes (different ordering, but same collection of pitches).



Transposition

A transposition of a piece of music is rewriting it in a different key. Using key signatures, this is a vertical translation along the staff. In other words, to transpose a piece of music to a new key, shift all of the notes up or down the same number of steps and then change the key signature. For example, to change a melody from C major to E^b major, each note needs to be raised a minor third, the interval between C and E^b. To accomplish the transposition, we add three flats to the key signature and then raise each note three steps on the staff (three because the interval is a third).

Exercise 4: Transpose the first four measures of *Twinkle Twinkle Little Star* from the key of C major to E major.

