Topics in Mathematics: Math and Music
Sections 2.2 and 2.3: Scales and Intervals

Scale	# of Notes	H-W Sequence						
Chromatic	12	All half steps						
Whone Tone	6	All whole steps						
Major	7	W	W	H	W	W	W	H
Natural Minor	7	W	H	W	W	H	W	W
Harmonic Minor	7	W	H	W	W	H	WH	H

Figure 1: The five types of scales discussed in Section 2.2 and their defining sequences of half (H) and whole (W) steps.

Examples
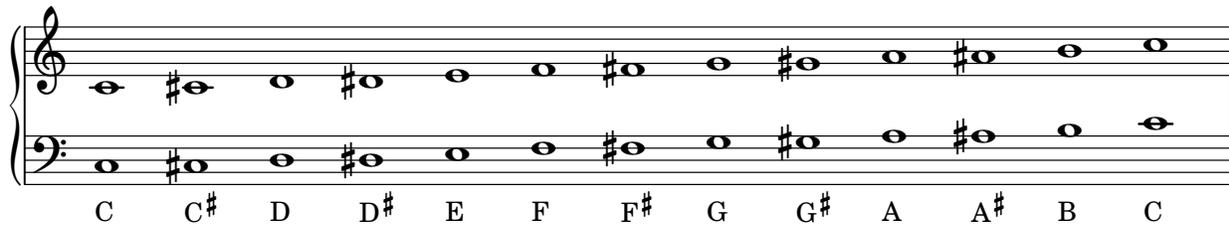
Figure 2: An ascending **chromatic scale** beginning on C. The scale has twelve notes, with consecutive notes always separated by a half step.
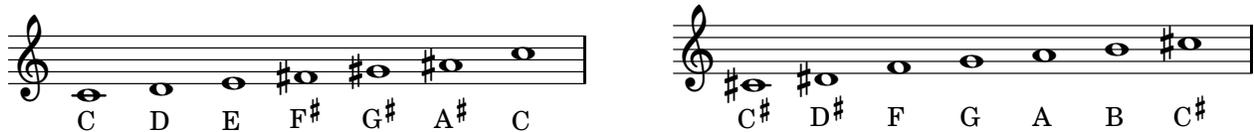
Figure 3: The two different **whole tone scales**, one beginning on C and the other on C$^\sharp$. In each case, the interval between consecutive notes is always equal to a whole step (two half steps).
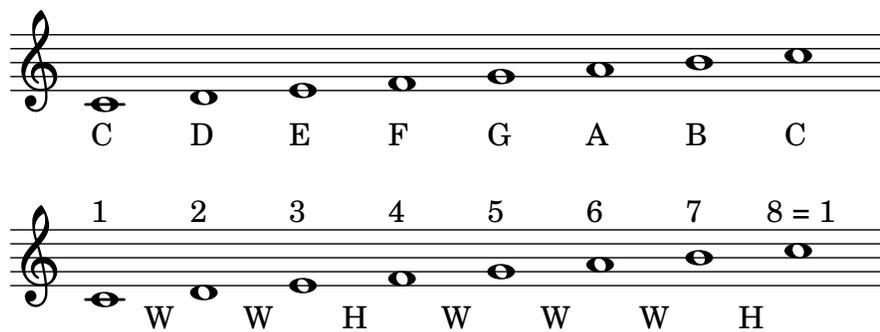
Figure 4: The **C major scale**. Notice the pattern of whole (W) and half (H) steps between consecutive notes. This same pattern holds for any major scale.
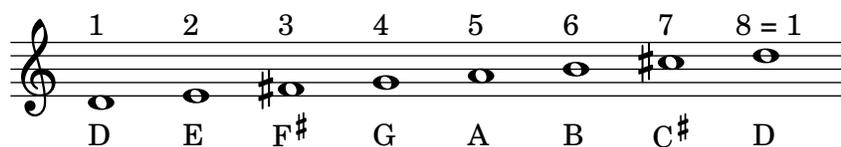
Figure 5: The **D major scale** in the treble clef.
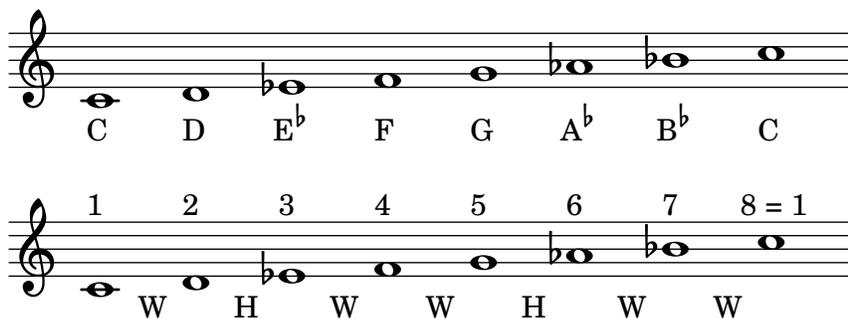
Figure 6: The **natural C minor scale**. Notice the pattern of whole (W) and half (H) steps between consecutive notes. This same pattern holds for any natural minor scale.
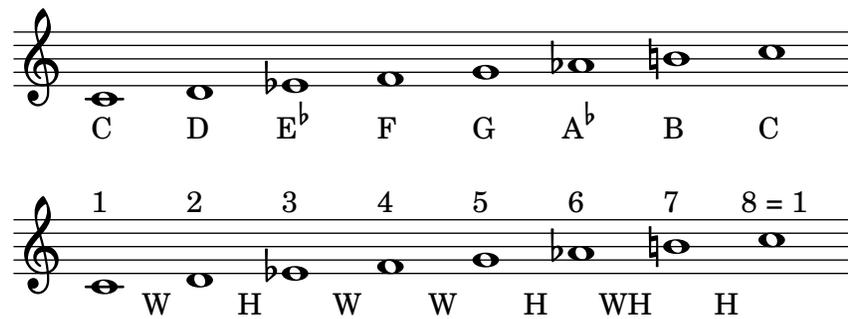
Figure 7: The **harmonic C minor scale**. The only difference from the natural C minor scale is that the seventh scale degree has been raised up a half step.

Intervals

Definition: Two notes are an <u>nth interval</u> apart if they are n steps from each other on the musical staff. When counting the number of steps, be sure to include the location of **both** the starting and ending note.

From a mathematical perspective, the definition of a musical interval is counter-intuitive because it over-counts the actual distance by one unit. The notes G and A are next to each other on the piano, so that should really be considered one step, not two. The notes A and C are two steps away on the piano keyboard, so that interval should really be a second, not a third. The musical version of measuring distance causes problems when combining intervals. For instance, we would expect that a second and a third combine to produce a fifth, but this is false! The interval from G to A (a second) plus the interval from A to C (a third) is equal to the interval from G to C (a fourth). According to music theory, the equation $2 + 3 = 4$ is valid!
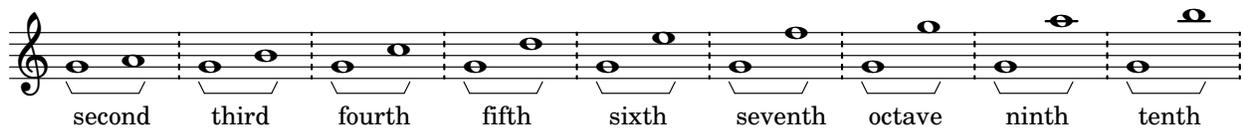
Figure 8: Some basic musical **intervals**. When counting the number of steps between two notes, always include the line or space of **both** the starting and ending note.

In addition to the number of steps in a musical interval, there is also the designation of **perfect**, **major** or **minor**, as well as augmented or diminished. The terms perfect and major are assigned to intervals corresponding to those found in the major scale. More specifically, if the major scale beginning on the bottom note contains the upper note as part of its scale, then the interval is perfect (in the case of the fourth or fifth) or it is major. The abbreviation for a major interval with n steps is Mn. Thus, M3 represents a major third and M6 means a major sixth. We use P4 and P5 to denote a perfect fourth and fifth, respectively.
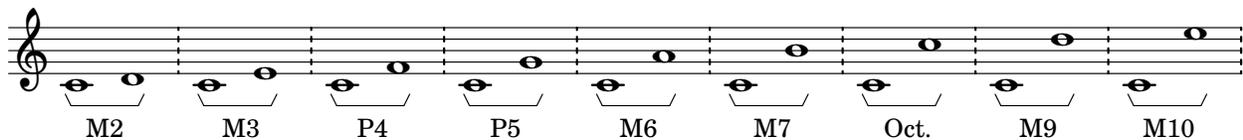
Figure 9: Some **major (M)** and **perfect (P)** intervals. In each case, the top note is a part of the C major scale.

If an interval is not perfect or major, we will consider it to be **minor**. The one exception is the interval that divides the octave in half, that is, an interval of six half steps, called a *tritone*. Some important minor intervals are shown in Figure 10, using middle C as the bottom note. Minor intervals are notated with a lower case m.
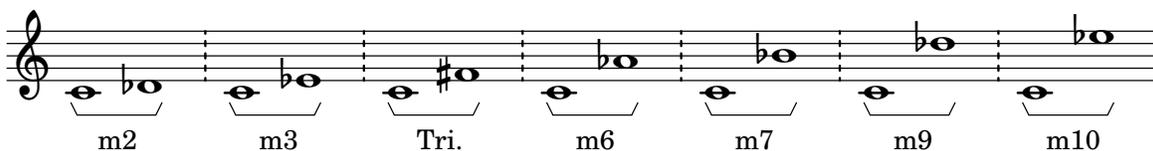
Figure 10: Some **minor (m)** intervals and the **tritone (Tri.)**.

Scale Deg.	Interval	Half Steps	Name	Musical Example(s)
1	Uni.	0	<u>Tonic</u>	
2	M2	2	Supertonic	*Frère Jacques* *Happy Birthday to You*
3	M3	4	Mediant	*Oh, When the Saints* *Kumbaya*
4	P4	5	<u>Subdominant</u>	*Here Comes the Bride* *Oh Christmas Tree*
5	P5	7	<u>Dominant</u>	*Twinkle Twinkle Little Star* *My Favorite Things*
6	M6	9	Submediant	*My Bonnie Lies over the Ocean* *It Came upon a Midnight Clear*
7	M7	11	<u>Leading tone</u>	*Take on Me*
8 = 1	Oct.	12	<u>Octave</u>	*Somewhere over the Rainbow*

Table 1: The **major (M)** and **perfect (P)** intervals within the octave, including the number of half steps in each interval, and some sample musical examples.

Notes	Interval	Half Steps	Musical Example(s)
C – D$^\flat$	m2	1	Theme from *Jaws*
C – E$^\flat$	m3	3	"Air-ball!" (the heckle interval) *Greensleeves*
C – F$^\sharp$	Tri.	6	*Maria* Theme from *The Simpsons*
C – A$^\flat$	m6	8	Theme from *Love Story* *Go Down Moses*
C – B$^\flat$	m7	10	*There's a Place for Us*

Table 2: The **tritone (Tri.)** and the **minor (m)** intervals within the octave, including the number of half steps in each interval, and some sample musical examples.

Exercises   (blank piano keyboard provided below for assistance)

1. Write an ascending whole tone scale (one octave) using only flats in the bass clef beginning on G. Use whole notes and write the letter name below each note.
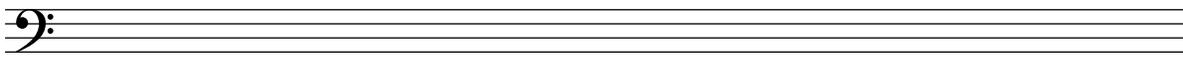
2. Write an ascending E♭ major scale (one octave) in the treble clef. Use whole notes and write the letter name below each note.
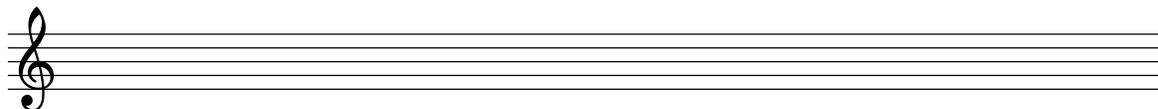
3. Write an ascending C♯ natural minor scale (one octave) in the treble clef. Use whole notes and write the letter name below each note.
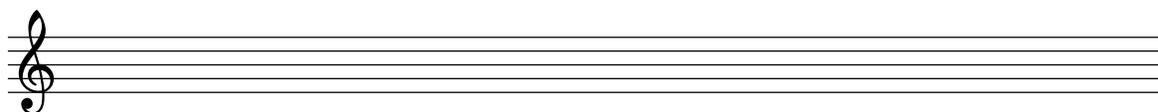
4. Write an ascending B♭ harmonic minor scale (one octave) in the bass clef. Use whole notes and write the letter name below each note.

5. Notate each of the following musical intervals, including major, minor and perfect designations.