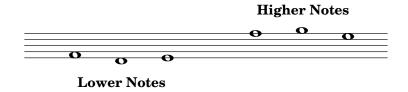
Topics in Mathematics: Mathematics and Music Section 2.1: Musical Notation

The Staff

Musical notes are situated on a set of five equally-spaced horizontal lines called a *staff*. The notes may be placed on the lines or in the spaces between lines. Lower notes are located at the bottom of the staff, while higher notes are found at the top.



Notes in Western music are named using a seven-letter alphabet:

A B C D E F G A B C D E F G ··· .

The process is *cyclic*, meaning that G is always followed by A, after which the list repeats again. The two notes above named "A" are an *octave* apart, a vitally important musical interval.

The Common Clefs



Figure 1: The various clefs used to indicate different notes on the staff. From left to right are shown the *treble*, *bass*, *alto* and *tenor* clefs.

The key to reading music in different clefs is to determine one particular note on the staff, and then use the cyclic musical alphabet to find the other notes. All clefs use the same seven letters, but the location of those particular notes varies from clef to clef. Climbing up one position on the staff, from line to space or vice-versa, corresponds to moving up one letter of the musical alphabet.

The notes on the staff for the treble clef are shown in Figure 2. Notice that the notes on the spaces of the treble clef spell out the word "FACE." A popular phrase to help recall the notes on the lines of the treble clef is "Every Good Boy Deserves Fudge," where the first letter of each word gives the names of the notes.



Figure 2: The notes on the staff for the treble or G clef. In this case, the note G is located on the line encircled by the spiral of the clef. As the notes move up the staff, the letter names move up the musical alphabet cyclically.

The bass clef or F clef always has its two dots surrounding the line corresponding to the note F. The full set of notes for the bass clef is shown in Figure 3. Common phrases for the notes on the lines and spaces of the treble clef are "Good Boys Deserve Fudge Always," and "All Cows Eat Grass," respectively.



Figure 3: The notes on the staff for the bass or F clef. Here, the note F is located on the line bounded by the two dots of the clef.

Ledger Lines

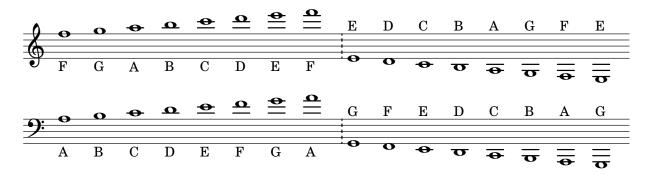


Figure 4: Ledger lines can be used to extend the staff above or below, in either the treble or bass clefs. Notes on and between ledger lines are named by following the musical alphabet cyclically.

Middle C



Figure 5: The location of middle C for the treble, bass, alto and tenor clefs (positioned from left to right, respectively).

The Piano Staff

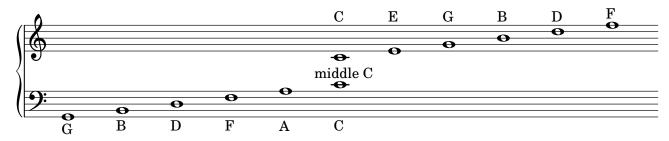


Figure 6: Climbing up the lines of the piano staff. The right hand reads the treble clef while the left hand follows the bass clef. The two clefs are connected by their common note, middle C.

The Piano Keyboard

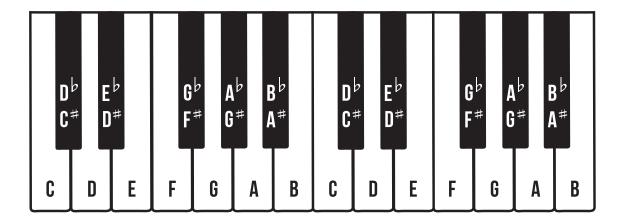


Figure 7: A section of the piano keyboard beginning on C. Note the different but equivalent names of the black keys. If we start on any particular note, the next time we encounter that same note we will have traveled an octave. **Memorize this figure!**

Accidentals

The basic building block in Western music is the *half step*, which corresponds to the interval between two consecutive keys on the piano. A \sharp raises the note by a half step while a \flat lowers it by a half step. To indicate a \sharp or \flat on the staff, the accidental is placed *before* the note it is intended to modify, on the *same* line or space as the note head.



Figure 8: Using sharps and flats to modify a note. The notes in each pair are identical, that is, they are enharmonically equivalent. The accidental is always placed before the note, and on the same line or space as the note head.

| $\underline{\text{Symbol}}$ | Name | $\underline{\text{Meaning}}$ |
|-----------------------------|--------------|-----------------------------------|
| # | Sharp | Raises the note by a half step |
| þ | Flat | Lowers the note by a half step |
| Ц | Natural sign | Cancels any previous accidental |
| х | Double Sharp | Raises the note by two half steps |
| bb | Double Flat | Lowers the note by two half steps |

Figure 9: The five different types of accidentals and their meaning.



Figure 10: Some music demonstrating the use of the natural sign (\$\bar{b}\$).

Exercises:

1. Write the name of each note (including accidentals) under each note.



2. Write the name of each note (including accidentals) under each note.



- 3. Take a good look at the piano keyboard in Figure 7. Now try and answer the following questions without using the keyboard. There is more than one possible answer to each question.
 - **a.** Name a note that is enharmonically equivalent to B^{\flat} .
 - **b.** Name a note that is enharmonically equivalent to G^{\sharp} .
 - **c.** Name a note that is enharmonically equivalent to $B^{\mathbf{X}}$.
 - **d.** Name a note that is enharmonically equivalent to F^{\flat} .
 - e. Name a note that is enharmonically equivalent to $E^{\flat\flat}$.
 - f. Name a note that is a half step above D.
 - **g.** Name a note that is a half step below F.
 - **h.** Name a note that is two half steps below C.
 - i. Name a note that is three half steps above E.
- 4. Using staff paper, write down all the names of the notes on the staff in the alto clef.
- 5. Using staff paper, write down all the names of the notes on the staff in the treble clef.