

Math and Music: Time Signatures

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Topics in Mathematics: Math and Music

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Section 1.2: Time Signatures

- A time signature consists of two numbers, kind of like a fraction. It appears at the start of a piece, although the time signature may change during a piece.
- The bottom number of a time signature indicates which type of note represents the **principal beat** in a measure, while the top number describes how many of these notes are required to fill one measure.
- The bottom number can be 1, 2, 4, 8, or 16, referring to a whole, half, quarter, eighth, or sixteenth note, respectively.
- Example: $\frac{9}{8}$ means that exactly nine eighth notes fill one measure.
- Example: $\frac{5}{2}$ means ?? Exactly five half notes fill one measure.

Time Signatures: Exercises

- 1 How many eighth notes are required to fill a measure in $\frac{5}{2}$ time?
Answer: $5 \cdot 4 = 20$ (4 eighth notes are equivalent to one half note, so 4 are needed to fill one beat)
- 2 How many sixteenth notes are needed to fill a measure in $\frac{5}{2}$ time?
Answer: $5 \cdot 8 = 40$ (8 sixteenth notes are equivalent to one half note, so 8 are needed to fill one beat; or double the previous answer)
- 3 How many quarter notes are required to fill a measure in $\frac{12}{8}$ time?
Answer: $12 \cdot \frac{1}{2} = 6$ (1 quarter note equals 2 eighth notes, so $1/2$ a quarter note equals one beat)

Sample Time Signatures

Different time signatures invoke different rhythmic styles.

- Most music (particularly pop music) is in $\frac{4}{4}$ time, denoted by **C** (common time).
- Marches are often in $\frac{2}{4}$ or $\frac{2}{2}$ time. So is the Latin dance style **Merengue**.
- Music in $\frac{3}{4}$ is well-suited to dances in three (e.g., waltz — oom-pah-pah oom-pah-pah).

Musical Example: $\frac{6}{8}$

Si - lent night, ho - ly night, All is calm, all is bright.

5 Round yon Vir - gin Moth - er and Child, Ho - ly In - fant so ten - der and mild,

9 Sleep in heav - en - ly peace, — sleep — in heav - en - ly peace.

The image shows a musical score for Franz Gruber's 'Silent Night' in 6/8 time. It consists of three staves of music in G major (one flat). The first staff contains the first four measures of the melody. The second staff starts at measure 5 and contains the next four measures. The third staff starts at measure 9 and contains the final four measures, ending with a double bar line. The lyrics are written below the notes, with hyphens indicating syllables that span across measures.

Figure: Franz Gruber's *Silent Night* (1818; text by Joseph Mohn), demonstrating $\frac{6}{8}$ meter, where there are 6 eighth notes per measure. Note the swaying, singsongy feel to the music. A measure of music in $\frac{6}{8}$ time is often subdivided into 2 parts (e.g., one-two-three two-two-three) so it can be felt in 2 or in 6.

Musical Example: $\frac{5}{4}$

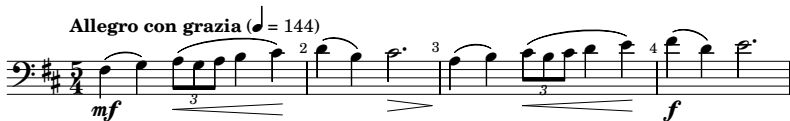


Figure: The opening melody (cellos) of the second movement of Tchaikovsky's *Symphony No. 6, "Pathétique"* (1893), a movement written entirely in $\frac{5}{4}$ meter. This was very rare for its time. The five beats in each measure can be grouped into a 2 + 3 pattern. The third beat of the first and third measures is called a **triplet**, where 3 eighth notes are compressed into the space normally occupied by 2.

Musical Example: Changing Meter

Tempo di Huapango (fast)

1 I like to be in A - mer - i - ca! 2 O. K. by me in A - mer - i - ca!

3 4

5 Ev-ry-thing free in A - mer - i - ca 6 7 8 For a small fee in A - mer - i - ca!

Figure: *America*, from Leonard Bernstein's *West Side Story* (1957), displaying a distinctive rhythmic pattern that alternates between $\frac{6}{8}$ (two primary beats per measure, each subdivided into three parts) and $\frac{3}{4}$ (three quarter-note beats, but the eighth-note pulse from the previous measure persists). The odd-numbered measures feel in 2 while the even numbered-measures are in 3. This is a musical depiction of the **commutative property**: $6 = 2 \cdot 3 = 3 \cdot 2$.

Guess the Time Signature

Try and determine the time signature of each piece.

- 1 Answer: $\frac{6}{8}$ *Everybody Hurts* by R.E.M. (1992)
- 2 Answer: $\frac{7}{4}$ *Solsbury Hill* by Peter Gabriel (1977). Note the 3 + 4 subdivision of each measure.
- 3 Answer: $\frac{5}{4}$ *Take Five* by Paul Desmond (1959). Note the 3 + 2 subdivision of each measure.



Rhythmic Repetition

A recurring rhythmic pattern is called an **ostinato**.



Figure: The opening of *Clocks* by Coldplay (2002) with its distinctive ostinato eighth-note pattern. Even though the time signature is $\frac{4}{4}$, each measure can be sub-divided into 3 + 3 + 2, giving it a more interesting, **syncopated** feel. This is actually a very famous subdivision common to music all over the world, such as bluegrass music, early rock and roll, and the **Charleston** dance rhythm.

Rhythmic Repetition: Salsa

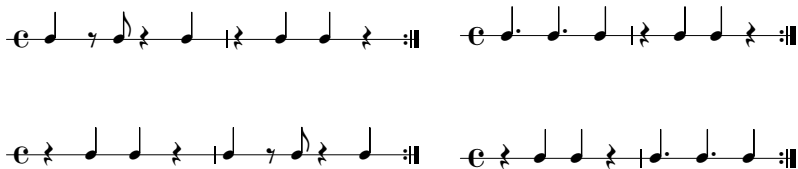


Figure: The 3-2 (*top*) and 2-3 (*bottom*) clave rhythmic patterns common in salsa music. Although notated differently, the left- and right-hand figures are rhythmically identical. The colon in front of the bar line at the end of the second measure is called a **repeat sign**, indicating that the two-measure pattern should be repeated.

Rhythmic Repetition: The Rite of Spring

Tempo giusto ($\text{♩} = 50$)

Violins *f*

Violas *f*

Cellos *f*

Basses *f*

5

Figure: The string parts in the opening eight bars of *Les Augures Printaniers: Danses des Adolescents* from Stravinsky's *Rite of Spring* (1913), featuring unexpected accents (indicated by $>$) to liven up the repeating eighth-note pattern.