MATH 136-03 Calculus 2, Spring 2019

Section 5.6: Net Change

This section focuses on **net change**, which is the total change in a quantity over a given time interval $a \leq t \leq b$. The key idea comes from the Fundamental Theorem of Calculus part 1: **the integral of a rate of change equals the net change.**

Suppose that $F(t)$ represents some quantity that is changing over an interval of time $a \leq t \leq b$, such as position of a vehicle or the amount of water in a tank. We would like to know the net change over that interval, defined as $F(b) - F(a)$. Since $F(t)$ is the antiderivative of $F'(t)$ by definition, we have the following interpretation of the Fundamental Theorem of Calculus part 1.

Net Change: The net change of $F(t)$ over the interval $[a, b]$ is found by integrating the rate of change $F'(t)$:

$$\int_a^b F'(t)\, dt \;=\; F(b) - F(a)\,.$$

Example 1: The number of cars per hour passing an observation point on a highway (called the **traffic flow rate**) is given by $r(t) = 1000 + 200t$, where $t = 0$ corresponds to 10 am (t is in hours).

(a) How many cars pass by between 10 am and 12 noon?

(b) How many cars pass by between noon and 3 pm?

Answer: Note that the units of $r(t)$ are cars per hour, which is a rate of change. Using the net change formula above, we have

$$\int_a^b r(t)\, dt \;=\; \text{number of cars after } b \text{ hours} - \text{number of cars after } a \text{ hours}\,.$$

Thus, to answer question **(a)**, we integrate $r(t)$ from $t = 0$ (10 am) to $t = 2$ (noon):

$$\int_0^2 1000 + 200t\, dt \;=\; 1000t + 100t^2\Big|_0^2 \;=\; 2000 + 400 - 0 \;=\; 2400 \text{ cars.}$$

For **(b)**, we integrate $r(t)$ from $t = 2$ (noon) to $t = 5$ (3 pm):

$$\int_2^5 1000 + 200t\, dt \;=\; 1000t + 100t^2\Big|_2^5 \;=\; 5000 + 2500 - (2000 + 400) \;=\; 5100 \text{ cars.}$$

Exercise 1: A population of rabbits grows at the rate of $10 + 4t + \frac{3}{5}t^2$ rabbits per week (t is in weeks). Find the number of rabbits after 5 weeks assuming there are 30 rabbits at time $t = 0$.

Next we consider the integral of velocity. Suppose that $s(t)$ is the position of an object at time t. As we know from Calc 1, the velocity is found by taking the derivative of position, $v(t) = s'(t)$. Thus,

$$\int_a^b v(t)\,dt \;=\; \int_a^b s'(t)\,dt \;=\; s(b) - s(a) \;=\; \textbf{net displacement} \text{ over } [a,b]\,.$$

However, if we wanted to compute the **total distance traveled**, we would need to take into account that the direction of motion could change (i.e., $v(t) > 0$ might mean traveling to the right, while $v(t) < 0$ means traveling to the left). For example, if we run 100 yards and then return back to our starting position, then our net displacement is zero ($s(b) = s(a)$), but the total distance traveled is 200 yards. To find how far we have traveled, we need to integrate the **speed** $|v(t)|$. This insures we are always integrating a positive rate of change.

Integral of Velocity: For an object in motion with velocity $v(t)$,

$$\int_a^b v(t)\,dt \;=\; \text{net displacement over } [a,b]$$

$$\int_a^b |v(t)|\,dt \;=\; \text{total distance traveled over } [a,b]\,.$$

Exercise 2: A particle has velocity $v(t) = 4t^2 - 28t + 40$ ft/sec. Find each of the following:

(a) the displacement over the interval $[0, 4]$,

(b) the total distance traveled over $[0, 4]$.