Principles of Analysis

MATH 242-01, MoTuTh 1:00 - 1:50, SWORDS 328, Fall 2009

Dr. Gareth Roberts

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Office Hours: Mon. 2:00 - 3:00, Wed. 1:00 - 3:00, Tues. & Thurs. 11:00 - 12:00 or by appointment.

Required Text: *Understanding Analysis*, Stephen Abbott

Web page: `http://mathcs.holycross.edu/~groberts/Courses/MA242/homepage.html` Homework assignments, schedule changes, exam materials, useful links and other important information will be posted at this site. Please bookmark it!

Prerequisites: MATH 241, Multivariable Calculus

Course objectives:

1. Develop critical thinking skills.
2. Read and write your own coherent and rigorous mathematical **proofs**.
3. Work and communicate with your peers.
4. Become an articulate and confident mathematician.

Syllabus: This course is designed for you to obtain a deeper understanding of the theoretical foundations of calculus and to introduce you to some of the more intriguing concepts in analysis. A major focus will be to think analytically and develop clear logical arguments which rigorously justify some of the major theorems from calculus. In other words, you will learn to write coherent mathematical PROOFS!

We will aim to cover Chapters 1-5 and Chapter 7 of the text. A tentative outline of the course is given below.

- Real Numbers, Axiom of Completeness, Bounds on Sets (4 classes)
- Cardinality, Countable and Uncountable Sets, Cantor's Theorem (3 classes)
- Sequences, Limits, Monotone Convergence Theorem (4 classes)
- Exam 1
- Subsequences, Bolzano-Weierstrass Theorem, Infinite Series (4 classes)
- Cantor Set, Open and Closed Sets, Compact Sets, Connected Sets (4 classes)
- Continuous Functions, Dirichlet's Function, Intermediate Value Theorem (6 classes)
- Exam 2
- Derivatives, Mean Value Theorem, L'Hôpital's Rule (4 classes)
- Riemann Integral, Fundamental Theorem of Calculus (5 classes)
- Final Exam

A Word of Caution: This course is substantially different from your previous math courses and you will often find it very challenging. Many problems will not have a single number or expression as an answer and learning a new technique to solve a problem will not usually be the focus. Gone are the days of rote procedures and mimicking example problems in the text.

However, most of the topics are certainly familiar to you — it's calculus. The difference here is the amount of rigor and analysis you will apply towards the subject. Evaluating a limit in calculus is usually straight-forward. Proving it rigorously from the ϵ - δ definition is much tougher.

We will focus on how to make a clear and precise argument as to why a particular statement is true. Beginning with a set of assumptions and building on other proven results, you will reason forth to the veracity of a claim. This is an acquired skill and one which will prove invaluable as you take more upper level math and science courses. I will try and allow you enough time and flexibility to develop this skill over the course of the semester. Hopefully, by the end of the semester you will have developed into a competent mathematician capable of proving theorems or conjuring up counterexamples to false claims.

Homework: There will be homework due nearly every Thursday at the START of class. Assignments will be posted on the course web page. There will be a list of problems for you to hand in, a nonempty subset of which will be graded. While you are allowed and encouraged to work on homework problems with your classmates, the solutions you turn in to be graded should be your own. No two proofs should look alike. Take care to write up your solutions **in your own words**. Plagiarism will not be tolerated and will be treated as a violation of the Departmental Policy on Academic Integrity.

NOTE: LATE homework will NOT be accepted. The only excused homework which is late will be accompanied by a letter from the Class Dean. However, you will be allowed ONE "mulligan" over the course of the semester where you can turn in the assignment up to one week after the original due date. No mulligans will be allowed on an assignment preceding a midterm exam.

Quizzes and Exams: There will be a weekly quiz given at the start of class every day a homework assignment is due. These will be short, definition/theorem quizzes to insure you are absorbing the material completely. The lowest quiz grade of the semester will be dropped. In addition, there will be 2 in-class exams and a comprehensive final at the end of the semester.

The exam schedule is given below. Please make a note of these dates and plan accordingly. Any conflicts must be legitimate and brought to my attention well before the exam is scheduled. If you have any specific learning disabilities or special needs and require accommodations, please let me know early in the semester so that your learning needs may be appropriately met. You will need to contact Dr. Neil Lipsitz in Disability Services (Hogan 209, x3693) to obtain documentation of your disability.

	Exam 1	**Thurs., Oct. 8**	**In Class**
Exam Schedule:	**Exam 2**	**Thurs., Nov. 19**	**In Class**
	Final	**Tue., Dec. 15**	**8:30 - 11:30 am**

Academic Integrity: The Department of Mathematics and Computer Science has recently drafted a policy on academic integrity to precisely state our expectations of students and faculty with regards to cheating, plagiarism, academic honesty, etc. You are required to read this policy and sign a pledge agreeing to uphold it. Anyone who violates the Departmental Policy on Academic Integrity will receive a 0 for that assignment as well as possible further disciplinary action involving your Class Dean.

Grade: Your course grade will be based on the scores you receive for each of the following items:

- classroom participation/interest 5%
- homework 25%
- quizzes 10%
- midterm exams 30%
- final exam 30%

How to do well in this course :

- ATTEND THE LECTURES, PARTICIPATE and ASK QUESTIONS.

  I take pride in my lectures and will work hard to get you to master the course material. However, this will not be of much use to you if you don't attend class. Furthermore, certain class periods will involve your participation in activities designed to get you to think. These days should be fun, with me lecturing little and you participating greatly. Do not take for granted the privilege you have of attending college. Value your time here and I will make it worth your while.

- DO YOUR HOMEWORK REGULARLY.

  The best way to learn mathematics is to *do* mathematics. This means mastering the material to the point where you could explain it to your classmates and your friends. "You don't really learn the subject until you teach it," is a common adage amongst mathematicians. It is not enough to know how to mimic an algorithm. A strong student should be able to follow and propose arguments as to why an algorithm is working or not working. This particularly applies to our course.

- WORK WITH YOUR CLASSMATES.

  Some of the best assets available to you are the knowledge and abilities of your peers. Learn to explain mathematics to your classmates. Mathematics can be fun and rewarding when there are people around you who enjoy figuring out problems as much as you do. Take advantage of this opportunity and organize study groups.

- READ THE TEXTBOOK.

  Yes, seriously, read the textbook frequently. The book was chosen in part for its readability and clear exposition. Concepts are motivated with interesting examples and placed in historical context. You can develop your proof techniques considerably by carefully following the proofs in the textbook line by line.