MATH 242: Principles of Analysis
Homework Assignment #1
DUE DATE: Thurs., Sept. 10, start of class.

Homework should be turned in at the BEGINNING OF CLASS. You should write up solutions neatly to all problems, making sure to show all your work. You are strongly encouraged to work on these problems with other classmates, although the solutions you turn in should be your **own** work. Please cite any references (web based or text) that you may have used for assistance with the assignment.

Note: Please list the names of any students or faculty who you worked with on the top of the assignment.

1. **a)** Prove that $\sqrt{3}$ is irrational.

   b) Prove that $\sqrt{2} + \sqrt{3}$ is irrational.

2. Let $f(x) = 1/x^2$.

   a) Find $f(A)$, where $A = \{x \in \mathbf{R} : 1 \le x \le 3\}$.

   b) Find $f(A)$, where $A = \{x \in \mathbf{R} : 0 < x\}$.

   c) Find $f^{-1}(B)$, where $B = \{x \in \mathbf{R} : 1 \le x < 4\}$.

   d) Find $f^{-1}(B)$, where $B = \{x \in \mathbf{R} : 0 < x < 1\}$.

3. Prove that for any function $f : A \to B$ and any subsets C and D of A, we have $f(C \cup D) = f(C) \cup f(D)$.

4. **a)** Prove that for any function $f : A \to B$ and any subsets C and D of A, we have $f(C \cap D) \subseteq f(C) \cap f(D)$.

   b) Find an example of a function $f : A \to B$ and subsets C and D of A such that $f(C \cap D) \neq f(C) \cap f(D)$.

5. Form the logical negation of each statement.

   a) $\forall x \in \mathbf{R}$, $\exists y \in \mathbf{R}$ such that $y^2 = x$.

   b) If $a \in A$, then $\sqrt{a}$ is an integer.

   c) Every student at Holy Cross eats breakfast at Kimball.

   d) Either Bob played soccer, or John did not play baseball.

6. Write the contrapositive and converse of each implication.

   a) If f is differentiable at $x = a$, then f is continuous at $x = a$.

   b) If the Yankees and Tigers win their respective divisions, then the Red Sox will make the playoffs.

7. Use induction to prove that $1 + 3 + 5 + \cdots + (2n - 1) = n^2 \ \ \forall n \in \mathbb{N}$.

8. Use induction to prove that $n^3 + 5n$ is an integer multiple of 6 $\ \forall n \in \mathbb{N}$.

9. Do the following exercise from the course text *Understanding Analysis* by Stephen Abbott: **1.2.3**.

   Hint: For problem **1.2.3 (c)**, try using the fact that $(A^C)^C = A$ for any set A along with the result from part **(b)**. Or you can prove it directly (the long way) by demonstrating inclusion in both directions.