MATH 242: Principles of Analysis
Homework Assignment #4
DUE DATE: Thurs., Oct. 1, start of class.

Homework should be turned in at the BEGINNING OF CLASS. You should write up solutions neatly to all problems, making sure to show all your work. You are strongly encouraged to work on these problems with other classmates, although the solutions you turn in should be your **own** work. Please cite any references (web based or text) that you may have used for assistance with the assignment.

Note: Please list the names of any students or faculty who you worked with on the top of the assignment.

1. Prove that the sequence $x_n = \frac{1}{8}\sin(\frac{n\pi}{2})$ diverges.

2. Prove that if the limit of a sequence exists, then it is unique. (See class notes from 9/24.)

3. Use the Big Limit Theorem to rigorously evaluate the following limits. You may utilize the fact that $\lim_{n\to\infty} 1/n^p = 0 \ \forall p \in \mathbb{R}$ satisfying $p > 0$.

   a) $\lim_{n\to\infty} \dfrac{7+3n-n^2}{5n^2+\sqrt{n}+1}$

   b) $\lim_{n\to\infty} \left(\dfrac{n^2}{3n^2+1}\right)^2$

4. Recall that the Fibonacci Series (Sequence) is defined by the recursive relation

$$F_{n+2} = F_{n+1} + F_n, \qquad F_1 = 1, F_2 = 1.$$

   Let $G_n = \dfrac{F_{n+1}}{F_n}$ be the sequence of ratios of consecutive Fibonacci numbers.

   a) Prove that $F_n F_{n+2} - F_{n+1}^2 = (-1)^{n+1} \ \forall n \in \mathbb{N}$.

   b) Prove that $\{G_{2n}\}_{n=1}^{\infty}$ is a decreasing sequence bounded below.

   c) Prove that $\{G_{2n-1}\}_{n=1}^{\infty}$ is an increasing sequence bounded above.

   d) Prove that the sequence G_n converges and find its limit. Why did I choose G for this sequence? (It is not because that's my first initial!)

5. Do the following exercises from the course text *Understanding Analysis* by Stephen Abbott: **2.3.3, 2.3.7(parts a) and b) only), 2.3.8, 2.4.2, 2.4.4**