Mathematical Models

MATH 303, Fall 2018

MWF 1:00 - 1:50, Swords 330

Professor Gareth E. Roberts

Contacting Me: Office: Haberlin 305, e-mail: `groberts@holycross.edu` phone: x2350

Office Hours: Mon., Wed., Fri. 2:00 - 3:00, Tues., 1:00 - 2:00, or by appointment.

Required Text: *Topics in Mathematical Modeling*, K. K. Tung, Princeton University Press (2007) (a copy is on reserve in the Science Library)

Other books on reserve in the Science Library:

1. *Mathematics and Climate*, Hans Kaper and Hans Engler, Society for Industrial and Applied Mathematics (2013)
2. *A First Course In Chaotic Dynamical Systems*, Robert L. Devaney (HC '69), Perseus Books Publishing (1992)

Course Prerequisites: MATH 241 (Multivariable Calculus) and MATH 244 (Linear Algebra)

Website: `http://mathcs.holycross.edu/~groberts/Courses/MA303/homepage.html`
Homework assignments, computer projects, lecture notes, useful links, and other important information will be posted at this site. Please bookmark it!

Course Objectives:

- Learn, apply, and synthesize some of the mathematical techniques used in math modeling.
- Develop skills to create, modify, and assess mathematical models.
- Construct, analyze, and evaluate your own mathematical model of a real world problem.
- Work and communicate with your peers.
- Have FUN applying your mathematical skills to the real world.

Syllabus: This course focuses on the construction, analysis, and interpretation of mathematical models designed to investigate real world problems and phenomena. Mathematical models have become increasingly important in understanding the world around us, particularly given the prevalence of data. For instance, controlled physical experiments of the Earth's climate are impossible; the only available approach is to use mathematical models, computer simulations, and data analysis.

This course is in the Applied Math breadth area and is inherently interdisciplinary. Examples will be drawn from fields such as biology, climate science, ecology, finance, medicine, physics, astronomy, chemistry, and sociology. The mathematical theory and techniques will be taught alongside the corresponding modeling application. These include concepts and tools from calculus, statistics, differential equations, and discrete dynamical systems. You will also be asked to work with real data, using Matlab and regression analysis to fit data to a curve and interpret the result.

Here is a sample list of the kinds of questions we will seek to answer using mathematical models:

- Why are the Fibonacci numbers so prevalent in the number of petals on a flower?
- How do biological properties of a mammal or bird, e.g., life span, metabolic rate, vary with its mass?
- How do I compute the monthly payment on my student loan?
- How old is the universe?
- What is the structure of the World Wide Web? Is there a mathematical law describing the network of links on the Internet?
- How many fishing licenses can we issue without decimating a particular fish population?
- What determines the Earth's surface temperature and how does it vary by latitude?
- What are the typical steady states for the Earth's climate system and how have these evolved over the history of the planet?
- What are "tipping points" and how do mathematicians study them?

We will cover material from Chapters 1–4, 6, and 8 of the primary text, with supplementary material provided as needed. We will also spend a few classes collaborating with visiting artists and scientists from the *Black Inscription* project, a special program sponsored by Arts Transcending Borders.

Specific topics to be explored, as well as special events, include:

- Fibonacci numbers and the golden ratio in nature and art
- Power (scaling) laws in biology and sociology
- Compound interest, mortgage payments, and discrete dynamical systems
- Modeling with differential equations: carbon dating, HIV modeling
- Qualitative approach to first-order ODE's, bifurcation theory
- Population models, logistic model, effects of harvesting
- Conceptual climate modeling, energy balance models, tipping points, Snowball Earth
- Climate and statistics, regression analysis, Keeling's CO_2 curve
- *Black Inscription* performance: 7:30 pm, Oct. 17 in The Pit (required)
- Field Trip to Broad Meadow Brook Conservation Center (required): analyzing water quality in a local brook, working with "messy" data

Homework: There will be several homework assignments (problem sets) given out during the course of the semester. Assignments will contain exercises from the textbook by K. K. Tung as well as some questions written by the instructor. All assignments will be posted on the course webpage. While you are allowed and encouraged to work on homework problems with your classmates, the solutions you turn in to be graded should be your own. If you must use the Internet for help on homework, be sure to cite the website(s) visited. Take care to write up solutions **in your own words**. Plagiarism will not be tolerated and will be treated as a violation of both the departmental policy on academic integrity and the college's policy on academic honesty.

NOTE: Late homework will not be accepted, particularly since students will be presenting solutions to certain homework problems on the day that they are due.

In-class Presentations: During the semester you will be expected to give a few in-class presentations demonstrating the solution to a particular homework problem (likely on the day the homework is due.) These presentations will be brief and are designed to enhance your oral communication skills as well as prepare you for your final project presentation.

Computer Projects: There will be 3–4 computer projects assigned over the course of the semester requiring the use of Matlab or some other type of mathematical software. These projects are numerical "experiments" designed to discover or reinforce important concepts discussed in class. One project will focus on analyzing data of CO_2 concentrations at Mauna Loa Observatory and bird flight speeds as a function of body mass. The goal will be to fit each data set to a curve, analyze the quality of the fits, and make predictions based on your models. Another project, inspired by the artists of *Black Inscription*, will explore creative ways to visualize and present data. Projects will be carried out in groups of 2 to 3 people, with one typed report to be turned in for the whole group.

Final Project: This course is designated as a "project course." You are required to complete a substantial final project (working in small groups) modeling some real world problem of your choice. The main goal is to identify an interesting problem of study, come up with a set of questions you would like to answer, collect the necessary data needed to explore your problem, formulate a model based on this data, analyze and refine your model, and finally, use your model to provide some answers to your initial questions. It is expected that you will adhere to the posted deadlines as your project develops (e.g., proposal, progress report, first draft). Your project will include both a written report and an in-class presentation during the final week of class.

Midterm Exam: There will be one midterm exam given in class on Friday, Oct. 5. If you have any specific learning disabilities or special needs and require accommodations, please let me know early in the semester so that your learning needs may be appropriately met. You will need to obtain approval from the Office of Disability Services (Hogan 215A, x3693).

Academic Integrity: The Department of Mathematics and Computer Science has drafted a policy on academic integrity to precisely state our expectations of both students and faculty with regards to cheating, plagiarism, academic honesty, etc. You are required to read this policy and sign a pledge agreeing to uphold it. A violation of the Departmental Policy on Academic Integrity will result in a 0 for that assignment and a letter describing the occurrence of academic dishonesty will be sent to your Class Dean.

Diversity and Inclusion: It is my intent that students from all diverse backgrounds and perspectives be well-served by this course, that students' learning needs be addressed both in and out of class, and that the diversity that students bring to this class be viewed as a resource, strength, and benefit. Any suggestions you have pertaining to diversity and inclusion are encouraged and appreciated.

Grade: Your course grade will be determined by the scores you receive for each of the following items:

- classroom participation/in-class presentations 10%
- homework and computer projects 40%
- midterm exam 20%
- final project 30%

How to do well in this course:

- Attend class, participate, and ask questions. Be an active learner.
- Do your homework regularly.
- Read the text. (Yes, this is possible!)
- Work with your classmates.

"Because mathematics is the language in which all things are quantified, it has a bearing on every aspects of our lives and cultures, whether we recognise it or not. Banking, security protocols, tsunami forecasting, medical scanning technology, airline scheduling and satellite navigation all have one thing in common: mathematics."

— John Toland (Director of the Isaac Newton Institute for Mathematical Sciences)

"Education is not the learning of facts, but the training of the mind to think."

— Albert Einstein