

Mathematical Models MATH 303

Homework Assignment #4

Due Fri., Oct. 26, 5:00 pm

You should write up solutions neatly to all problems, making sure to show all your work. A nonempty subset will be graded. You are encouraged to work on these problems with other classmates, and it is ok to use internet sources for help if it's absolutely necessary (with proper citation); however, the solutions you turn in should be your own work and written in your own words.

Note: Please list the names of any students or faculty who you worked with on the assignment.

1. Read Chapter 4, "Differential Equation Models: Carbon Dating, Age of the Universe, HIV Modeling," from the course textbook *Topics in Mathematical Modeling* by K. K. Tung.

Complete the following exercises from Chapter 4 (pp. 79–83): **1, 2, 3, 4**

2. Consider the differential equation $\frac{dy}{dt} = 3y^4 - 12y^2$.

- (a) Sketch the phase line and identify each equilibrium point as a sink, source, or node.
- (b) On the same set of coordinate axes, plot four solutions $y(t)$ obtained from the following initial conditions: $y(0) = -3$, $y(0) = -1$, $y(0) = 1$, and $y(0) = 3$.

3. Consider the differential equation $\frac{dy}{dt} = Ay - y^3$, where $A \in \mathbb{R}$ is a parameter.

- (a) Find the equilibrium points in terms of A .
- (b) For what value(s) of A does a bifurcation occur? Describe in words the qualitative behavior of solutions (e.g., number and type of equilibria) before, at, and after the bifurcation.
- (c) Sketch the bifurcation diagram for varying A , including several different phase lines. Plot A on the horizontal axis and y on the vertical axis. Stable equilibria should be drawn as a solid curve, while unstable equilibria should be dashed.