Final Project
Dynamical Systems, MATH 374
Fall 2017

The last assignment of the course is to complete a final project focusing on some particular aspect or application related to the course material. Your project will consist of both a typed report (approximately 10 pages) and an in-class presentation (15–20 minutes) during the final week of class. Your report can be written using Maple (which has nice word processing for mathematical symbols), LaTex (if you know it), or a regular word-processing program with a hand-written appendix for mathematical calculations. You are required to work in a small group (2–4 people), although it is expected that each member of the group will contribute equally. Your group does not have to be identical to the one used on the computer projects. The final project is worth 25% of your total course grade.

Timeline and Due Dates:

- Oct. 31: Brief description of final project topic due, including at least three references.
- Nov. 21: Brief progress report (1–2 typed pages) due detailing the status of your project, including results and further lines of inquiry. By this date you should have met with me at least once to discuss the content of your project and the plan for your presentation.
- Dec. 1, 5pm: Detailed outline of your report and presentation due, including title and a list of all group members.
- Dec. 5 and 7: Project Presentations (each member must speak for at least 5 minutes).
- Dec. 8, 5pm: Final Report due (typed, roughly 10 pages).

Some Sample Topic Suggestions:

The aim of this project is for you to explore a specific topic in dynamical systems or some application that uses the theory we have studied. Ideally, you will apply mathematical knowledge gained from this course, as well as others, to make an in-depth investigation of your topic. This can involve reading research papers or textbooks and presenting the results, as well as doing some actual mathematics. For example, if a paper you are reading states that a period-doubling bifurcation occurs at a particular parameter value, then you should verify this. This is not expected to be a ground-breaking research paper leading to publication, but rather a chance for you to delve deeper into a topic employing your well-developed mathematical abilities.

Some sample topics are described below, along with a few possible resources. Feel free to suggest your own topic if there is something different you would like to investigate.

Caution: Be careful when using material found on the Internet. For example, some of the information on *Wikipedia* is correct and some is not. Be sure to check your findings thoroughly by confirming them with at least two independent, published (i.e., peer-reviewed) sources. In general, your sources should be scholarly articles or books. It is recommended that you utilize the powerful search engine MathSciNet (linked from the course webpage) to help locate relevant source materials.

1. **Newton's Method** There are some equations that computers have a lot of trouble solving. For example, when applying Newton's method to certain polynomials, the iterative method may actually fail to find a root for an entire open set of initial guesses. Such bad polynomials are interesting to study from a dynamical systems perspective. Interesting fractals arise from applying Newton's method to complex polynomials.

   Sample Resources: Chapter 13 of Devaney's text and "Newton's Versus Halley's Method: A Dynamical Systems Approach," G. E. Roberts and J. Horgan-Kobelski, *International Journal of Bifurcation and Chaos*, Vol. 14, No. 10 (2004), 3459–3475.

2. **A Queueing Model** How do we model the decision-making process? Suppose that you have two tasks that must be completed in a certain amount of time. While you do one job, the other waits in the queue. There are some surprisingly complicated dynamics that can arise from such a simple system. Connections to the subject of economic dynamics are revealed in the nice paper by James Walsh titled, "Surprising Dynamics From a Simple Model," *Mathematics Magazine*, Vol. 79, No. 5 (2006), 327–339.

3. **Period Three Implies Chaos** Study the famous paper that ushered in the era of Chaos. What are the main theorems? How are they proven? Compare and contrast the proofs with those we did in class. "Period Three Implies Chaos," T. Y. Li and J. A. Yorke, *American Mathematical Monthly*, Vol. 82, No. 10 (1975), 985–992.

4. **Applications to Population Dynamics** Where have biologists and scientists who try to model real-world populations needed dynamical systems theory? Find a particular model with parameters and study the bifurcations and dynamical behavior. What are the implications of the theory to the fate of the populations being studied?

   Sample Resources: "Biological Populations with Nonoverlapping Generations: Stable Points, Stable Cycles, and Chaos," R. M. May, *Science*, Vol. 186, (1974), 645-647; "Simple Mathematical Models with Very Complicated Dynamics," R. M. May, *Nature*, Vol. 261, June 10, 1976, 459–467.

5. **Applications to Medicine** How has the field of dynamical systems and chaos theory influenced medicine? Can fractal dimension help us better diagnose skin cancer? Give some specific examples, exploring the dynamical phenomena that occur.

   Sample Resources: "Fractal Physiology and Chaos in Medicine," B. West, Studies of Nonlinear Phenomena in Life Science, 1, World Sci. Publ., Teaneck, NJ, 1990; "Dynamical Disease—the Impact of Nonlinear Dynamics and Chaos on Cardiology and Medicine," L. Glass, The Impact of Chaos on Science and Society (Tokyo, 1991), 219–231, United Nations Univ. Press, Tokyo, 1997; "Nearly One-Dimensional Dynamics in an Epidemic," W. M. Schaffer and M. Kot, *Journal of Theoretical Biology*, Vol. 112 (1985), 403–427; "Fractals and Cancer," J. W. Baish and R. K. Jain, *Cancer Research*, Vol. 60, 3683–3688.

6. **Applications to Economics** How has chaos theory influenced economics? Can we really use fractals to better understand the price of a given commodity or the ups and downs of the financial markets? Give specific examples including the relevant topics from dynamical systems being used.

   Sample Resources: "Chaos and Chaotic Dynamics in Economics," M. Faggini, *Nonlinear Dyn. Psychol. Life Sci.*, Vol. 13, No. 3, (2009), 327–340; *Chaos and Nonlinear Models in Economics: Theory and Applications*, J. Creedy and V. L. Martin, Edward Elgar Publishing (1994); "Chaos and Nonlinear Forecastability in Economics and Finance," B. LeBaron, *Philos. Trans. Roy. Soc. London Ser. A*, Vol. 348, No. 1688 (1994), 397–404; "A Multifractal Walk Down Wall Street," B. Mandelbrot, *Scientific American* (1999), 70–73.

7. **Fractal Geometry** How can fractals help deepen our understanding of the natural world? Where and how have the ideas of fractal geometry been successfully applied in other disciplines? What are the different ways to determine the fractal dimension of a set?

   Sample Resources: *The Fractal Geometry of Nature*, B. Mandelbrot, W. H. Freeman and Company, New York, 1983; "A Brief Historical Introduction to Fractals and Fractal Geometry," L. Debnath, *Internat. J. Math. Ed. Sci. Tech.*, Vol. 37, No. 1 (2006), 29–50; "Selected Topics in Mathematics, Physics, and Finance Originating in Fractal Geometry," B. Mandelbrot, Thinking in Patterns, 1–33, World Sci. Publ., River Edge, NJ, 2004.

8. **Number Theory and Dynamical Systems** There are many interesting dynamical systems that arise in the field of number theory. These are sometimes grouped into a field called arithmetic dynamics. One nice example is the Ducci game given by the map

   $$D(x_1, x_2, \ldots, x_n) \;=\; (|x_1 - x_2|, |x_2 - x_3|, \ldots, |x_n - x_1|),$$

   where $x_i \in \mathbb{Z}$ or $x_i \in \mathbb{Z}_m$. Study this map dynamically. What are the periodic points? What is the fate of most orbits? What are the open questions concerning this map?

   Sample Resources: "The N-number Ducci Game," M. Chamberland and D. Thomas, *J. Difference Equ. Appl.*, Vol. 10, No. 3 (2004), 339–342; "A Characterization for the Length of Cycles of the N-Number Ducci Game," N. Calkin, J. G. Stevens, D. Thomas, *Fibonacci Quart.*, Vol. 43, No. 1 (2005), 53–59; "Periods in Ducci's N-Number Game of Differences," A. Ehrlich, *Fibonacci Quart.*, Vol. 33, No. 4 (1990), 302–305.

9. **Fractals and Dynamics in the Arts** There are many examples of artists and composers using concepts from dynamical systems in their creations. For example, the Hungarian composer György Ligeti (1923–2006) mimicked the butterfly effect in his piano piece *Désordre* ("Disorder," 1985), by making a slight change to a repeating rhythmic pattern in one hand. As with the butterfly effect, this small change grows to have a dramatic impact on the music. Find some other examples in art and/or music and explain the connection to dynamical systems.

   Sample Resources: "A Dynamical Systems Perspective on Music," D. Burrows, *J. Musicology*, Vol. 15 (1997), 529–545; *African Fractals*, R. Eglash, Rutgers University Press, 1999. Also see my lecture notes and sources for my Montserrat course *Math/Music: Aesthetic Links* at
   `http://mathcs.holycross.edu/~groberts/Courses/Mont2/homepage.html`