Dynamical Systems

MATH 374, Fall 2017

TuTh 12:30 - 1:45, Smith Labs 155

Professor Gareth E. Roberts

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Office hours: Mon. 10:30 - 12:00, Tues. 2:00 - 3:00, Wed. 11:00 - 12:00, Thurs. 10:00 - 11:00, or by appointment.

Required Texts: *A First Course In Chaotic Dynamical Systems*, Robert L. Devaney (HC '69) and *Chaos: Making a New Science* (20th-anniversary edition), James Gleick

Course Prerequisite: MATH 242

Web page: `http://mathcs.holycross.edu/~groberts/Courses/MA374/homepage.html`
Homework assignments, computer projects, exam materials, useful links and other important information will be posted at this site. Please bookmark it!

Course objectives:

- Learn, apply, and synthesize the mathematical techniques of dynamical systems.
- Become proficient at making clear and coherent mathematical arguments.
- Work and communicate with your peers.
- Have FUN learning dynamical systems!

Syllabus: The goal of this course is to study the mathematical theory and applications of the fascinating field of dynamical systems. Any process that changes over time can be viewed as a dynamical system. The weather, the amount of money in your bank account, the population of north Atlantic right whales, and the motion of the planets are all examples of processes that can be modeled as dynamical systems.

We will restrict our attention to discrete dynamical systems as opposed to continuous ones (modeled by differential equations), the key concept being the **iteration** of a function. Specifically, given a function $f : X \mapsto X$ mapping some space X to itself and an initial seed x_0 in X, we are interested in following the **orbit** of x_0 under iteration of f. The orbit of x_0 is the sequence of numbers

$$x_0,\ f(x_0),\ f(f(x_0)),\ f(f(f(x_0))), \dots$$

For a simple example, typing a number and repeatedly hitting the sine key on a calculator gives the orbit of that number under the sine function.

There are two overarching themes that will guide our study. The first aim is to classify the underlying dynamical behavior of a given system. What types of orbits exist? Are there periodic orbits? What are they and how many? Are there "dense" orbits? Is it possible to describe the fate of all orbits? What is the structure of the set of points where the "interesting" dynamical behavior occurs? The second goal is to consider a family of functions depending on a parameter and study the changes in dynamical behavior as the parameter varies. This has particular relevance to real-world systems since measurements in the field are always approximations. What dynamical structures persist? What are the key values where the dynamical behavior changes substantially?

By what mechanisms do these changes occur? For these types of questions we will investigate some very famous bifurcation diagrams.

Our study of dynamical systems will be approached from an analytic, graphical, and numerical viewpoint. You will quickly see how different fields of mathematics are all incorporated in the study of dynamical systems. There will be computational exercises as well as proofs. Numerical work will be conducted through various computer projects. This course is in the Applied Math breadth area; while some applications will be discussed in class, you will explore a specific application in great detail by completing a final project.

We will cover most of the material in the text except for Chapters 13 and 18. A rough outline of the semester is as follows:

- Sample dynamical systems and iteration (1 class)
- Orbits: fixed points, periodic points, dense orbits, attractors and repellers (2 classes)
- Visualizing orbits: graphical analysis, histograms (2 classes)
- Bifurcations: saddle-node, period doubling, orbit diagrams, the quadratic family (2 classes)
- Exam I (Chapters 1 through 6)
- Topological conjugacy: symbolic dynamics, shift map (4 classes)
- Chaos: chaotic dynamical systems, sensitive dependence on initial conditions, Feigenbaum's constant (4 classes)
- Sarkovskii's theorem, the Schwarzian derivative (2 classes)
- Exam II (Chapters 7 through 12)
- Fractals: Sierpinski triangle, Koch's snowflake curve, dimension (2 classes)
- Complex dynamical systems: Julia sets, the Mandelbrot set (3 classes)
- Final Project presentations (2 classes)

Homework: There will be 6 – 7 homework assignments given out during the course of the semester. Assignments will be posted on the course web page. There will be a list of problems for you to hand in, a nonempty subset of which will be graded. While you are allowed and encouraged to work on homework problems with your classmates, the solutions you turn in to be graded should be your own. Take care to write up solutions **in your own words**. Plagiarism will not be tolerated and will be treated as a violation of both the departmental policy on academic integrity and the college's policy on academic honesty.

NOTE: LATE homework will NOT be accepted. However, you will be allowed ONE "mulligan" over the course of the semester where you can turn in the assignment up to one week after the original due date.

Computer Projects: There will be 3 – 4 computer projects assigned over the course of the semester using Maple and/or software that is available on the web at `http://math.bu.edu/DYSYS/applets/` This website contains java applets from the Dynamical Systems and Technology Project at Boston University overseen by the author of the primary course text, Robert Devaney. The projects are numerical "experiments" designed to discover or reinforce important concepts in the theory of dynamical systems. One project will involve a competition to see which group can compute the most digits of a famous mathematical constant. Projects will be carried out in groups of 2 to 3 people with one typed set of solutions to be turned in for the whole group.

Final Project: You are required to complete a substantial final project (working in small groups) focusing on some particular aspect or application related to the course material. Details and suggestions of topics will be distributed early in the semester. You may find a topic of interest as you read James Gleick's book *Chaos*. Your project will include both a written report and an in-class presentation during the final week of class.

Exams: There will be two midterm exams given in class. Please mark these dates down and plan accordingly. Any conflicts must be legitimate and brought to my attention well before the exam is scheduled.

If you have any specific learning disabilities or special needs and require accommodations, please let me know early in the semester so that your learning needs may be appropriately met. You will need to obtain approval from the Office of Disability Services (Hogan 215A, x3693).

Exam Schedule:

Exam 1	**Thurs., Oct. 5**	**In Class**
Exam 2	**Thurs., Nov. 16**	**In Class**

Academic Integrity: The Department of Mathematics and Computer Science has drafted a policy on academic integrity to precisely state our expectations of both students and faculty with regards to cheating, plagiarism, academic honesty, etc. You are required to read this policy and sign a pledge agreeing to uphold it. A violation of the Departmental Policy on Academic Integrity will result in a 0 for that assignment or exam, and a letter describing the occurrence of academic dishonesty will be sent to your Class Dean.

Diversity and Inclusion: It is my intent that students from all diverse backgrounds and perspectives be well-served by this course, that students' learning needs be addressed both in and out of class, and that the diversity that students bring to this class be viewed as a resource, strength, and benefit. Any suggestions you have pertaining to diversity and inclusion are encouraged and appreciated.

Grade: Your course grade will be determined by the scores you receive for each of the following items:

- classroom participation/interest/effort 5%
- homework and computer projects 30%
- midterm exams 40%
- final project 25%

How to do well in this course:

- Attend class, participate and ask questions.
- Be an active, responsible learner.
- Do your homework regularly.
- Read the texts. (Yes, this is possible!)
- Work with your classmates.