MATH 392: Seminar in Celestial Mechanics
Homework Assignment #2
DUE DATE: Thurs., Jan. 31, start of class.

Homework should be turned in at the BEGINNING OF CLASS. You should write up solutions neatly to all problems, making sure to show all your work. You are strongly encouraged to work on these problems with other classmates, although the solutions you turn in should be your **own** work. Please cite any references (web based or text) that you may have used for assistance with the assignment.

Note: Please list the names of any students or faculty who you worked with on the top of the assignment.

1. Read Chapter 2 of *Celestial Mechanics: The Waltz of the Planets*, by Celletti and Perozzi. List a few of Henri Poincaré's contributions to the field of Celestial Mechanics. What is the great, outstanding conjecture made by Poincaré concerning solutions to the n-body problem? *Note:* This is not the famous Poincaré Conjecture of topology recently proven by reclusive Russian mathematician Grigori Perelman for which he will receive (or refuse) a million-dollar prize from the Clay Mathematics Institute should his proof be verified.

2. Prove the vector identity
$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$$
used to derive the integration constant $\mu\mathbf{e}$ in class.

3. (Hard) Suppose that $\mathbf{q}(\mathbf{0}) \times \mathbf{v}(\mathbf{0}) = \mathbf{c} \neq \mathbf{0}$ and let $c = ||\mathbf{c}||$. Show that there is an orthogonal matrix A and a change of variables $\mathbf{x} = A\mathbf{q}$ such that the central force problem is converted to
$$\ddot{\mathbf{x}} = -\frac{f(r)}{r}\,\mathbf{x}$$
where $r = ||\mathbf{x}||$ and the new angular momentum is simply $(0, 0, c)$. The hard part is constructing the matrix A. You may need to review some of your linear algebra.

4. Recall the key equation for the Kepler problem derived in class
$$r = \frac{c^2/\mu}{1 + e\cos\gamma} \tag{1}$$

   a) Show that in polar coordinates (r, γ), equation (1) determines an ellipse if $0 < e < 1$, a parabola if $e = 1$ and a hyperbola if $e > 1$. *Hint:* By treating the eccentricity vector $\mathbf{e}$ as the positive x-axis, convert the equation into rectangular coordinates using $x = r\cos\gamma$ and $y = r\sin\gamma$.

   b) For the case $e = 1$, which way does the parabola open? What are the rectangular coordinates of the vertex?

   c) For the case $0 < e < 1$ or $e > 1$, what are the rectangular coordinates of the center?

d) For the case of the hyperbola $e > 1$, show that the eccentricity e satisfies the equation

$$\frac{b}{a} = \sqrt{e^2 - 1}$$

where a and b are the lengths of the semi-major and semi-minor axes, respectively. What happens to the shape of the hyperbola as the eccentricity increases?

5. Do the following exercises from Pollard's text: **4.1, 5.3, 5.4**.

   Some hints: For **4.1**, you can either use your rectangular coordinate version of equation (1) or use the geometric definition of the given conic section. Problems **5.3** and **5.4** are straightforward by utilizing the correct formulae.