MATH 392: Seminar in Celestial Mechanics

Homework Assignment #4

DUE DATE: Thurs., Feb. 14, start of class.

Homework should be turned in at the BEGINNING OF CLASS. You should write up solutions neatly to all problems, making sure to show all your work. You are strongly encouraged to work on these problems with other classmates, although the solutions you turn in should be your **own** work. Please cite any references (web based or text) that you may have used for assistance with the assignment.

Note: Please list the names of any students or faculty who you worked with on the top of the assignment.

1. Read Chapter 4 of *Celestial Mechanics: The Waltz of the Planets*, by Celletti and Perozzi. Explain why we are only able to see roughly 60% of the Moon's surface from the Earth. Why is it not exactly 50%?

2. (Courtesy of James, Alex, Suzy and Ashley:) Derive the quadratic formula by completing the square in the equation
$$ax^2 + bx + c = 0.$$

3. Consider the Kepler problem and assume that the initial position $\mathbf{q_0}$ is nonzero.

   a) Show that the motion is circular if and only if $\mathbf{q}(t)$ and $\mathbf{v}(t)$ are orthogonal for all time t.

   b) For strictly elliptic motion ($0 < e < 1$), suppose that at some time t^*, the position $\mathbf{q}(t^*)$ and velocity $\mathbf{v}(t^*)$ are orthogonal. Show that the planet is located at either pericenter or apocenter at time t^*.

   c) Suppose that at time $t = 0$, the initial position and velocity are given by the vectors $\mathbf{q}_0 = (x_0, y_0, 0)$ and $\mathbf{v}_0 = \lambda(-y_0, x_0, 0)$, respectively, where $\lambda \geq 0$ is a scalar parameter. Draw a bifurcation diagram (eg. a λ number-line) describing the different types of motion possible as λ varies. Where do the bifurcations occur and what is the corresponding motion?

   d) In the previous question, there are two open intervals corresponding to strictly elliptic (non-circular) motion. What is the difference between each case? Sketch a sample orbit and give the time t of pericenter for each case.

4. Consider the Kepler problem with gravitational constant $\mu = 9$ and mass $m = 1$. Suppose that at time $t = 0$, a planet has initial position and velocity given by the vectors $\mathbf{q}_0 = (-4, 2, 4)$ and $\mathbf{v}_0 = (0, 1, -1)$, respectively.

   a) Find the values of h, e, c and a.

   b) Find the vectors $\mathbf{c}$ and $\mathbf{e}$.

   c) At what time does the planet pass through pericenter? What is the period of the orbit?

   d) Find the precise location (in rectangular coordinates) of the planet at time $t = 5$. This may require the use of a computer algebra system such as Maple.

5. Prove that the eccentric anomaly u is precisely the angle PCS in Figure 3 on pg. 25 of Pollard's text.

6. Do the following exercises from Pollard's text: **10.2, 14.2**.

   Some hints: For **10.2**, first describe a point on the ellipse in terms of a, b, e and u. Figure 3 on pg. 25 should be helpful. Then use two trig. identities for the difference of two cosines and the difference of two sines to simplify the distance Q_0Q_1 to the given formula. For **14.2**, try setting $r(t) = \alpha|t - t_1|^k$ and use the given limit to obtain conditions on α and k.