MATH 392: Seminar in Celestial Mechanics
Homework Assignment #5
DUE DATE: Thurs., Feb. 21, start of class.

Homework should be turned in at the BEGINNING OF CLASS. You should write up solutions neatly to all problems, making sure to show all your work. You are strongly encouraged to work on these problems with other classmates, although the solutions you turn in should be your **own** work. Please cite any references (web based or text) that you may have used for assistance with the assignment.

Note: Please list the names of any students or faculty who you worked with on the top of the assignment.

1. Read Chapter 5 "Our chaotic Solar System" of *Celestial Mechanics: The Waltz of the Planets*, by Celletti and Perozzi.

   a) Give a few examples of how "chaos" arises in celestial mechanics.

   b) What is the Titius-Bode law? Where is it accurate and where does it fail? How did it help lead to the discovery of the main asteroid belt? *Side Note:* This could make for an interesting final project examining/deriving other possible "laws" and comparing and contrasting them with this one.

2. **Center of Mass:** The point of this problem is to show that the formula for the center of mass in the two-body problem matches up with our intuitive, physical understanding of the concept. Recall that the center of mass is defined as

$$\mathbf{q}_c = \frac{m_1\mathbf{q}_1 + m_2\mathbf{q}_2}{m_1 + m_2}$$

   a) By rewriting $\mathbf{q}_c$ in a clever way, show that the head of the vector $\mathbf{q}_c$ lies on the line segment between the two bodies at $\mathbf{q}_1$ and $\mathbf{q}_2$. Assume that the tails of the vectors $\mathbf{q}_c$, $\mathbf{q}_1$ and $\mathbf{q}_2$ are identical (at the origin is fine.)

   b) Show that if $m_1 > m_2$, then the center of mass $\mathbf{q}_c$ is located closer to $\mathbf{q}_1$ than $\mathbf{q}_2$. Similarly, show that if $m_2 > m_1$, then the center of mass $\mathbf{q}_c$ is located closer to $\mathbf{q}_2$ than $\mathbf{q}_1$. Finally, if $m_1 = m_2$, show that the center of mass is equidistant between the two bodies.

3. Show that the differential equation for $\mathbf{Q}_2$ becomes

$$\ddot{\mathbf{Q}}_2 = -\frac{Gm_1^3}{(m_1 + m_2)^2} \cdot \frac{\mathbf{Q}_2}{||\mathbf{Q}_2||^3}$$

   when converting the 2-body problem into barycentric coordinates.

4. Show that the various energies (kinetic, potential and total) taken with respect to the center of mass (assumed to be at the origin) split between the bodies in the ratio m_2/m_1. In other words, show that

$$\frac{U_1}{U_2} = \frac{K_1}{K_2} = \frac{h_1}{h_2} = \frac{m_2}{m_1}$$

5. Do the following exercises from Pollard's text: **Chapter 2: 1.2, 1.3**.

   Some hints: In problem #1.3, the new law of attraction should be the same as Newton's except that r_{jk}^{-2} in Equation (1.1) is replaced by r_{jk}. Then you should do three things: First, show that the center of mass can be moved to the origin by changing variables as usual (be sure to show that the new center of mass is really at the origin.) Second, simplify the new equations of motion and show that they decouple. Third, solve the resulting system writing your solution in terms of the initial conditions of the n-body problem (initial positions and velocities) as well as any other relevant constants.