MATH 392: Seminar in Celestial Mechanics
Homework Assignment #8
DUE DATE: Thurs., March 27, start of class.

Homework should be turned in at the BEGINNING OF CLASS. You should write up solutions neatly to all problems, making sure to show all your work. You are strongly encouraged to work on these problems with other classmates, although the solutions you turn in should be your **own** work. Please cite any references (web based or text) that you may have used for assistance with the assignment.

Note: Please list the names of any students or faculty who you worked with on the top of the assignment.

1. Read Chapter 8 "Rock around the planets" of *Celestial Mechanics: The Waltz of the Planets*, by Celletti and Perozzi. What are **halo orbits** and where do they get their name? Do they have any practical significance for spacecraft missions?

2. Consider the collinear Kepler problem where the center of attraction is at the origin and the axis of motion is taken to be the positive x-axis. Using this set up, the equation of motion is simply
$$\ddot{x} = -\frac{\mu}{x^2}$$
where $\mu > 0$ is the gravitational constant and $x > 0$ is assumed.

   a) Convert the second-order equation into a planar first-order system and show that it is Hamiltonian. What is the Hamiltonian function H?

   b) Sketch the complete phase portrait of the system.

   c) In terms of the energy H and initial velocity v_0, what conditions will ensure the solution ends in a collision?

   d) Suppose the body starts from rest $v_0 = 0$ with initial position $x(0) = x_0$. Show that collision will occur at time
$$T = \frac{\pi}{\sqrt{\mu}} \left(\frac{x_0}{2}\right)^{3/2}$$

   e) Use the information from part **d)** to solve **Ex. 1.1** in Chapter 2 of Pollard (from HW #7).

3. Recall that the Lagrange multiplier ω^2 for a relative equilibrium $\mathbf{x}$ can be found via $\omega^2 = U(\mathbf{x})/(2I(\mathbf{x}))$. Show that this formula gives the correct value computed in class for the equilateral triangle solution of Lagrange.

4. Show that in the spatial 4-body problem, placing the bodies at the vertices of a regular tetrahedron yields a central configuration, regardless of the masses. *Hint:* Use the mutual distances as coordinates.

5. **a)** Suppose that we are given a planar central configuration $\mathbf{x}$ in the n-body problem, all of whose bodies lie on a circle centered at the origin of radius r. Suppose that the center of mass of the configuration is at the origin. Show that it is possible to add another body at the origin of **arbitrary** mass m to obtain a planar c.c. in the $(n+1)$-body problem. What is the new Lagrange multiplier ω^2_{new} in terms of the old one ω^2?

   b) Use part **a)** to show three bodies of equal mass placed at the vertices of an equilateral triangle and a fourth body of arbitrary mass placed at the center of the triangle yields a planar c.c. in the 4-body problem.