Math, Music and Identity

Worksheet: Sir Peter Maxwell Davies and Magic Squares

Definition: An $n \times n$ square array containing the numbers

$$1, \, 2, \, 3, \, 4, \, \cdots, \, n^2 - 1, \, n^2,$$

each occurring precisely once, and having each row, column, and main diagonal sum to the same amount, is called a *magic square*.

8	1	6
3	5	7
4	9	2

Table 1: The *Lo Shu* magic square, known to the ancient Chinese over 3,000 years ago. Each row, column, and main diagonal sum to 15.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

Table 2: The 4×4 magic square used by Dürer in his famous engraving *Melancholia I*. Each row, column, and main diagonal sum to 34. The date of the carving, 1514, can be found in the middle of the last row of the magic square.

Magic Squares Fun Fact: The rows, columns, and main diagonals of any $n \times n$ magic square always sum up to the same value, the *magic constant*. This value is given by the formula

$$M_n = \frac{n(n^2 + 1)}{2}.$$

n	2	3	4	5	6	7	8	9	10
M_n	5	15	34	65	111	175	260	369	505

Table 3: The magic constant M_n for different values of n.

A Mirror of Whitening Light, Peter Maxwell Davies (Opus #75; 1977)

C	A	A$\sharp$	F$\sharp$	D	D	C$\sharp$	G
G$\sharp$	F	F$\sharp$	D$\sharp$	G	D	C$\sharp$	C$\sharp$
A	G$\flat$	G	E	A$\flat$(D)	E$\flat$	D	D (A$\flat$)
G	D$\sharp$	E	C$\sharp$	A (D)	G$\sharp$	G	D (A)
B	B	C	F	C$\sharp$	E	E$\flat$	F$\sharp$
F	B$\flat$	B	C	E	G	F$\sharp$	B$\flat$
E	A	A$\sharp$	B	D$\sharp$	F$\sharp$	F	A
F	E	F	B	G	A	G$\sharp$	C

Table 4: Davies' special matrix of notes derived from a plainchant melody, transposition of that melody, and the 8 × 8 magic square of Mercury. The notes in parentheses are adjustments to the square that were used by the composer. The notes and durations for the piece come from following different patterns through this special musical magic square.

Figure 1: The flute part starting at rehearsal letter Z in *A Mirror of Whitening Light*.

8	2	3	5	4	6	7	1
1	7	6	4	5	3	2	8
1	7	6	4	5 (8)	3	2	8 (5)
8	2	3	5	4 (1)	6	7	1 (4)
8	2	3	5	4	6	7	1
1	7	6	4	5	3	2	8
1	7	6	4	5	3	2	8
8	2	3	5	4	6	7	1

Table 5: The 8×8 magic square of Mercury reduced modulo 8, except that any number equivalent to 0, that is, any multiple of 8, has been written as an 8.

Exercises:

1. By writing the specific letter names of the notes and their durations (count by eighth notes and include rests), confirm that the flute part shown in Figure 1 corresponds to a counterclockwise spiral through Davies' magic square for the notes (Table 4) and their durations (Table 5).

2. Figure 2 shows the first violin part for *A Mirror of Whitening Light*, beginning at rehearsal letter F1.

   a. List the letter names of the notes consecutively and find the pattern in Table 4 used by Davies to construct this sequence of notes.

   b. Consider the durations of each note, including rests. Find the pattern in the reduced magic square (Table 5) corresponding to the lengths of each note. For the entire excerpt, count using the quarter note as one beat rather than an eighth note.

Figure 2: The first violin part in Davies' *A Mirror of Whitening Light*, starting at rehearsal letter F1.

3

3. Complete the following 4×4 magic square.

	13	11	
7			1
14			12
	8	2	

4. Complete the following 6×6 magic square.

	1	6		19	24
3	32	7	21		25
31		2		27	20
8	28	33		10	
30	5	34	12		16
4	36		13	18	

5. Prove that the sum of the corners in any 4×4 magic square is also equal to 34, the magic constant.