Math, Music and Identity

Some Examples of Symmetry in Music

1. **Transposition** (Vertical Translation)

   - Stadium sports chants (organ)
   - *Symphony No. 5 in C minor*, Ludwig Van Beethoven (see textbook, Figure 5.10)
   - *Stars and Stripes Forever*, John Phillip Sousa
   - *Adagio for Strings*, Samuel Barber (see textbook, Figure 5.11)
   - *You Raise Me Up*, Secret Garden

2. **Retrograde** (Vertical Reflection)

   - *Lean on Me*, Bill Withers
   - The *Crab Canon* from Johann Sebastian Bach's *Musical Offering* (see textbook, Figure 5.13)
   - *Piano Sonata No. 26, "Minuet and Trio"* in A major, Joseph Haydn (see textbook, Figure 5.15). Similar material appears in Haydn's *Sonata No. 4 for Piano and Violin* and his *Symphony No. 47 in G major*, "The Palindrome."
   - *Hallelujah Chorus* from the *Messiah*, George Frideric Handel (music accompanying the text "for the Lord God Omnipotent reigneth"; see attached excerpt).

3. **Inversion** (Horizontal Reflection)

   - Most of Bach's fugues have a *subject* (a short musical theme) that returns later in inverted form. Example: Bach's *Fugue No. 8 in D♯ minor* from the *Well-Tempered Clavier*, vol. I (see textbook, Figure 5.5 and attached music).
   - *Subject and Reflection*, No. 141, Volume 6 of *Mikrokosmos*, Béla Bartók (see textbook, Figures 5.7 and 5.8).
   - *Contrapunctus XI*, Johann Sebastian Bach (see textbook, Figure 5.9)
   - Opening of the march *The Thunderer*, John Philip Sousa (see textbook, Figure 5.17)

4. **Retrograde-inversion** (180° Rotation)

   - *Ludus Tonalis* ("Game of Tones"), Paul Hindemith. The final movement is equivalent to the first movement rotated by 180° (see textbook, Figure 5.16).

5. **Symmetries appearing simultaneously**

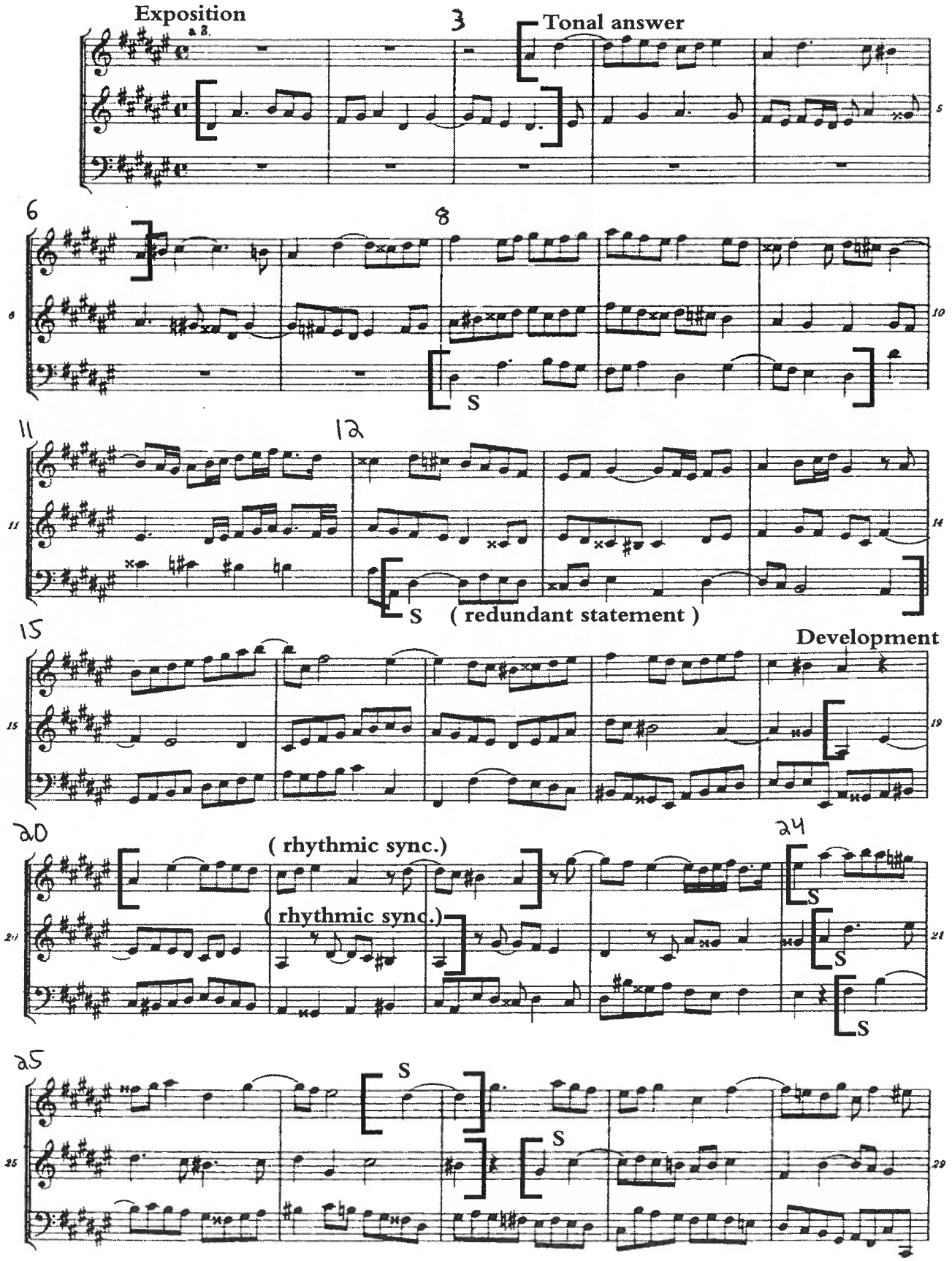
   - *Játékok for piano I, Hommage à Eőtvős Péter*, György Kurtág, (see attached excerpt). This piece is symmetric under *both* a horizontal and a vertical reflection, and therefore also has 180° rotational symmetry.

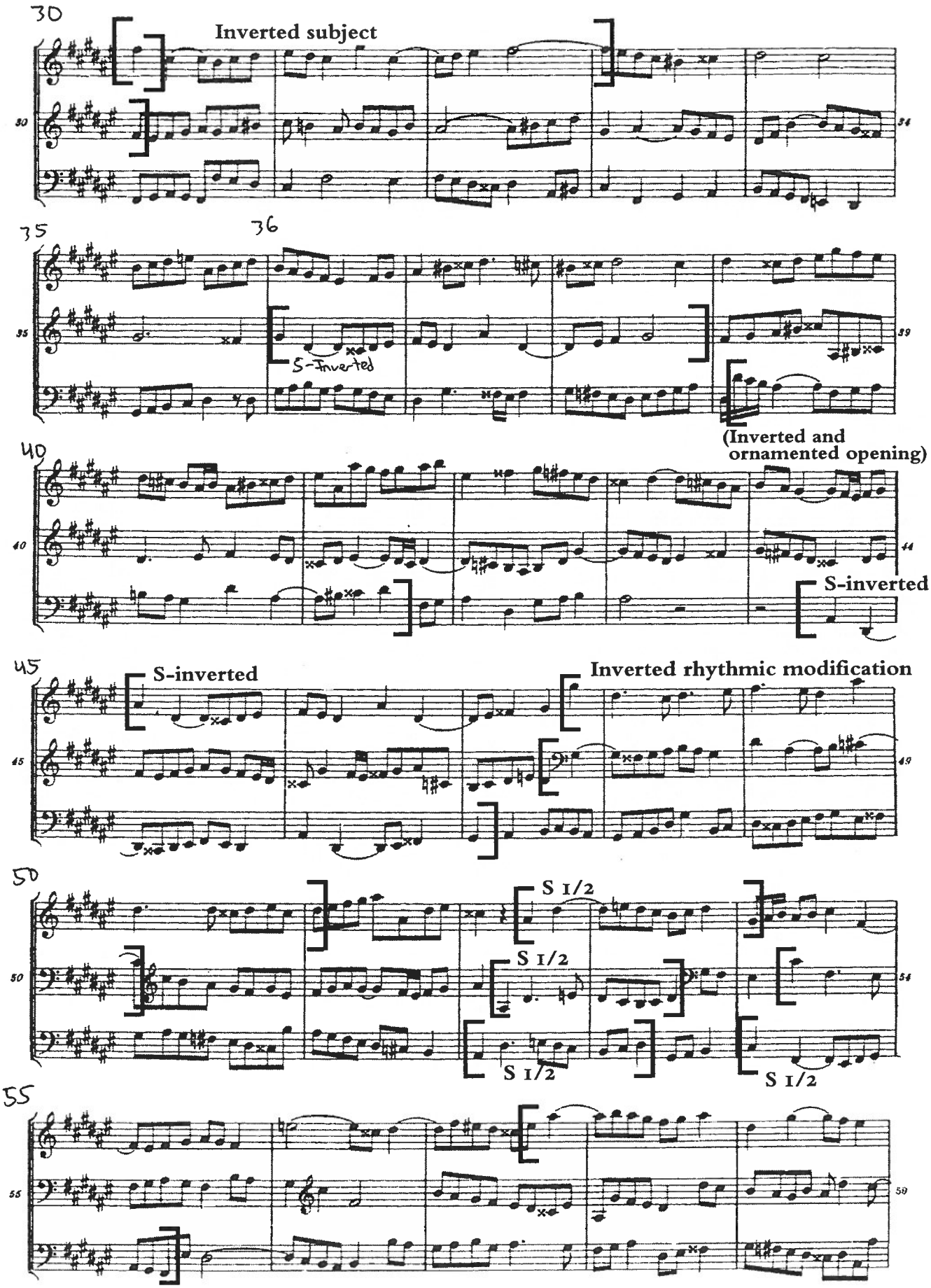
6. **Pieces featuring symmetry types 1, 2 and 3**

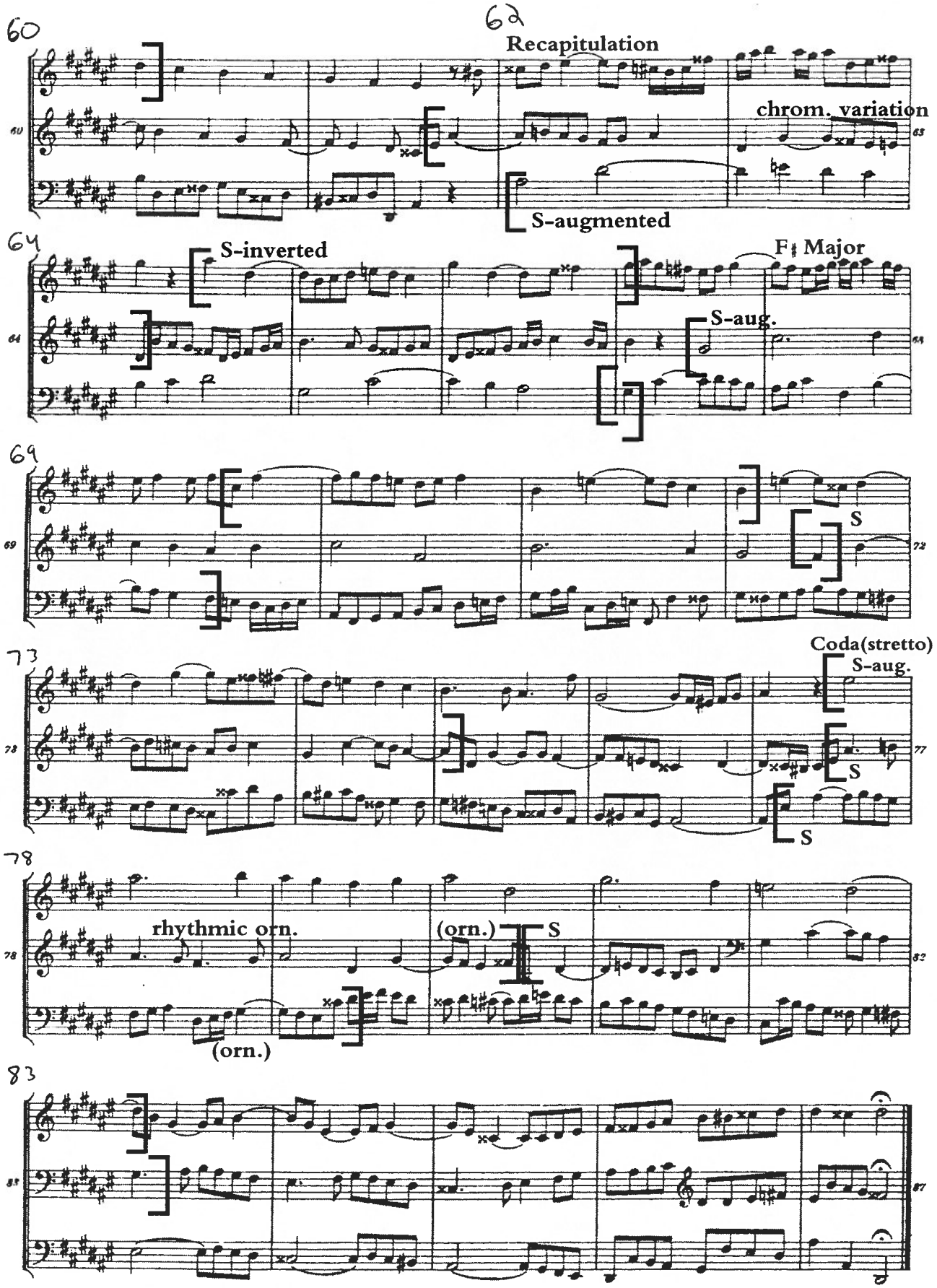
   - Bach's *Musical Offering*
   - Hindemith's *Ludus Tonalis*
   - Developmental section of *Hungarian Rhapsody #2*, Franz Liszt (see attached excerpt)
   - *I Got Rhythm*, George Gershwin (a bit of a stretch; see attached music)
   - Many examples in the style of Schoenberg's *Twelve-Tone Method* (see Chapter 7 of the textbook).

Example 5.2 *Fugue No. 8 in d♯ minor from WTC I by J.S. Bach*

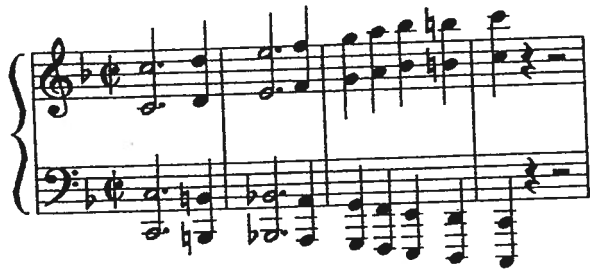
ABA' form (Sonata form)




30
Inverted subject
35
36
S-Inverted
(Inverted and ornamented opening)
40
S-inverted
45
S-inverted
Inverted rhythmic modification
50
S 1/2
S 1/2
S 1/2
S 1/2
55


60
62
Recapitulation
chrom. variation
S-augmented
64
S-inverted
F♯ Major
S-aug.
69
S
73
Coda(stretto)
S-aug.
S
S
78
rhythmic orn.
(orn.)
S
(orn.)
83

John Philip Sousa "The Thunderer"



Franz Liszt "Hungarian Rhapsody # 2"

THE MATH BEHIND THE MUSIC

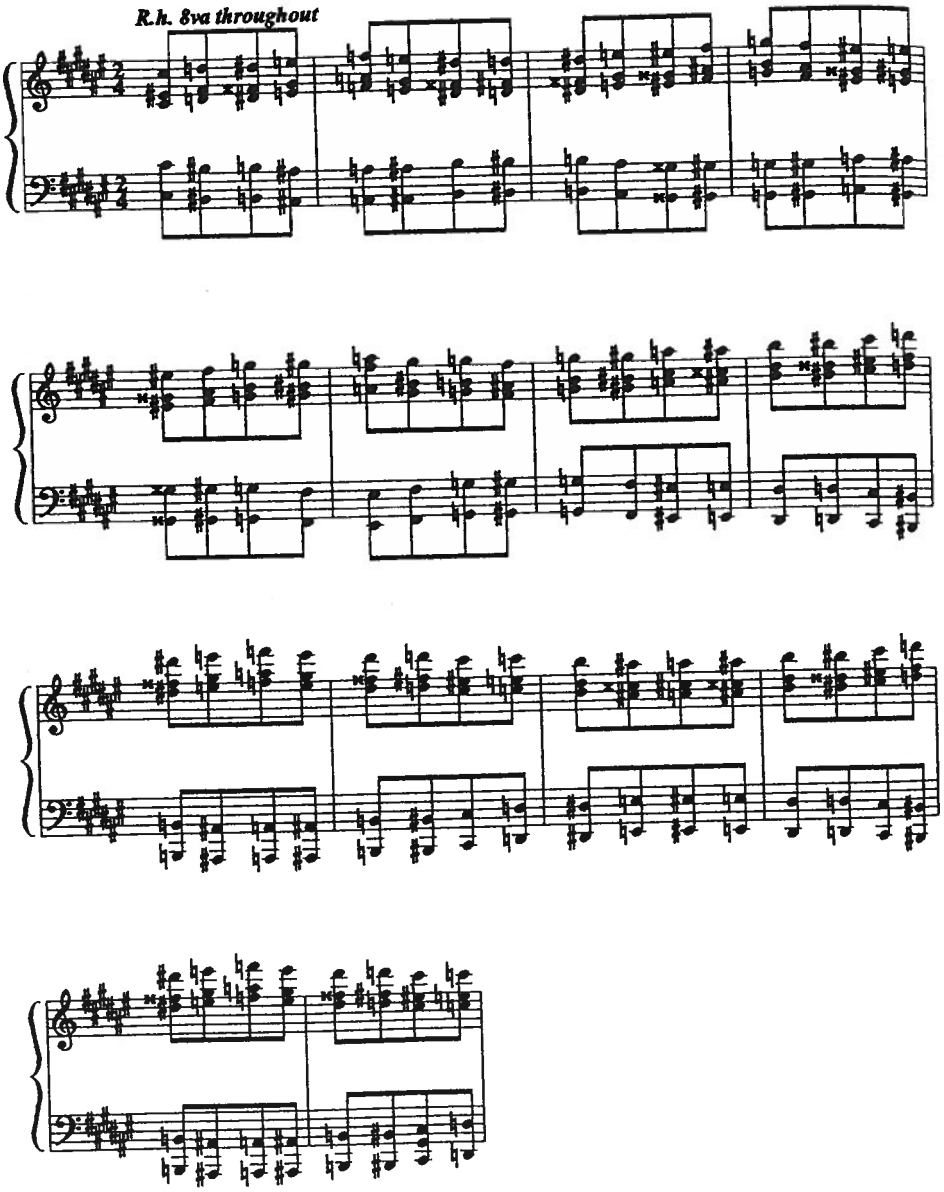
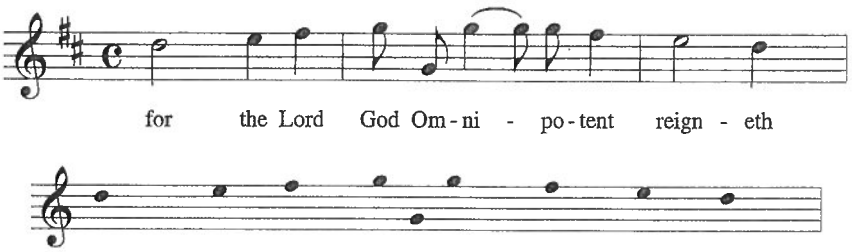
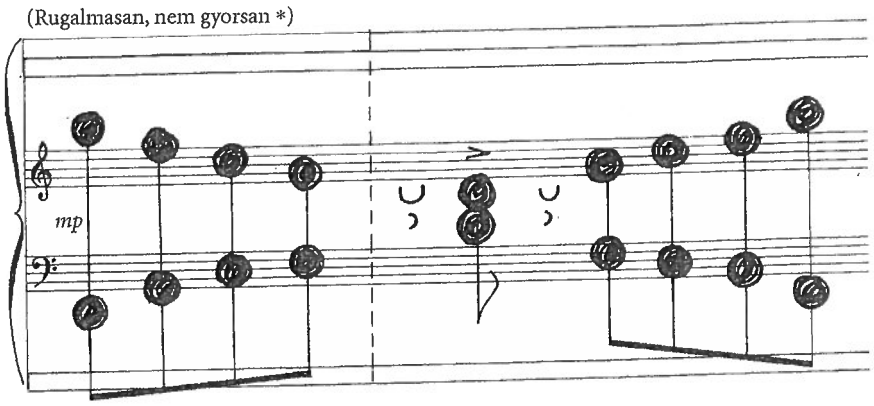


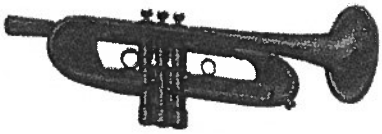
Fig. 11



ge F. Handel, *Messiah, Hallelujah chorus.*



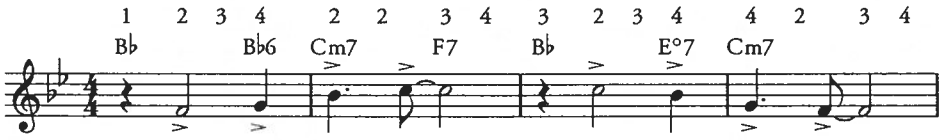
rg Kurtág, *Játékok for piano I, Hommage à*
ős Péter.



Thirty-two-Bar Song

The next form we're going to look at is the *thirty-two-bar song form.* A great example of that is a George Gershwin song entitled "I Got Rhythm." A bar indicates the way the beats are divided. For example, a song in 4/4 time, like "I Got Rhythm," has four beats in each bar (SCORE ILLUS. 9a AND b). Because the form of the song is made up of four equal sections of eight bars each, it is known as a thirty-two-bar song. Eight times four equals thirty-two.

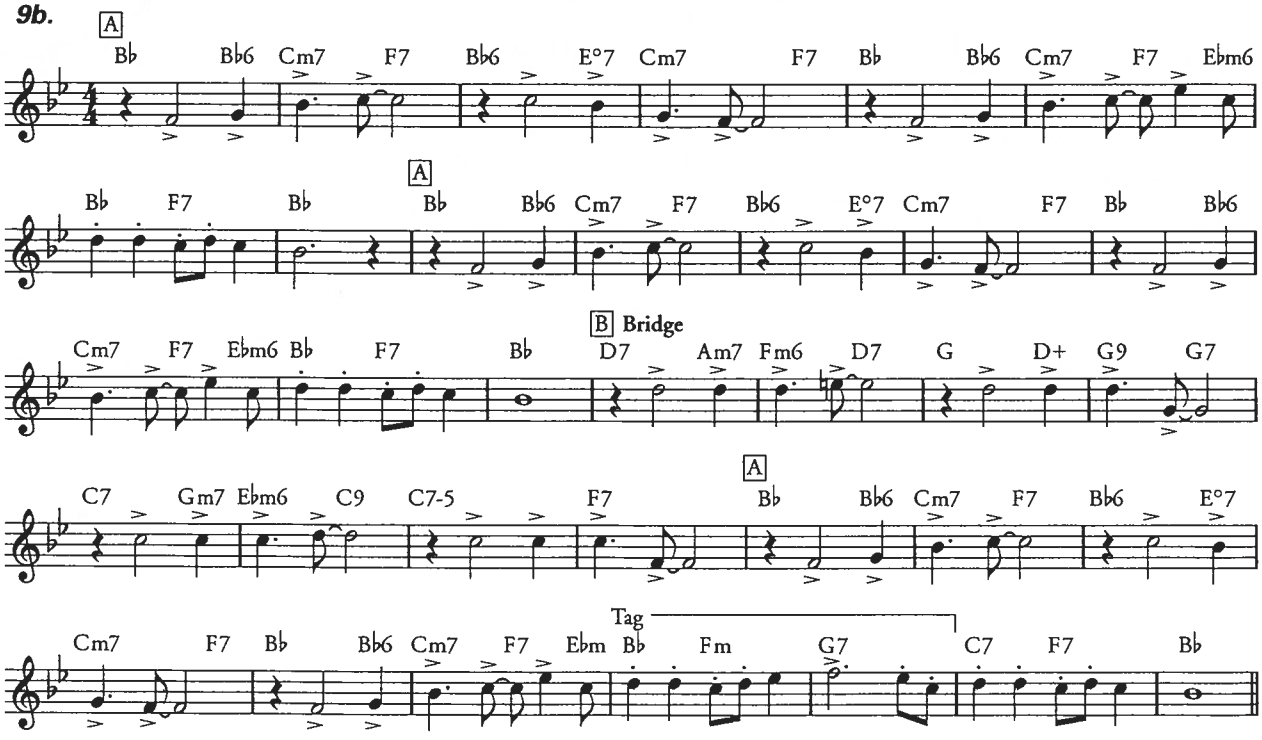
9a.



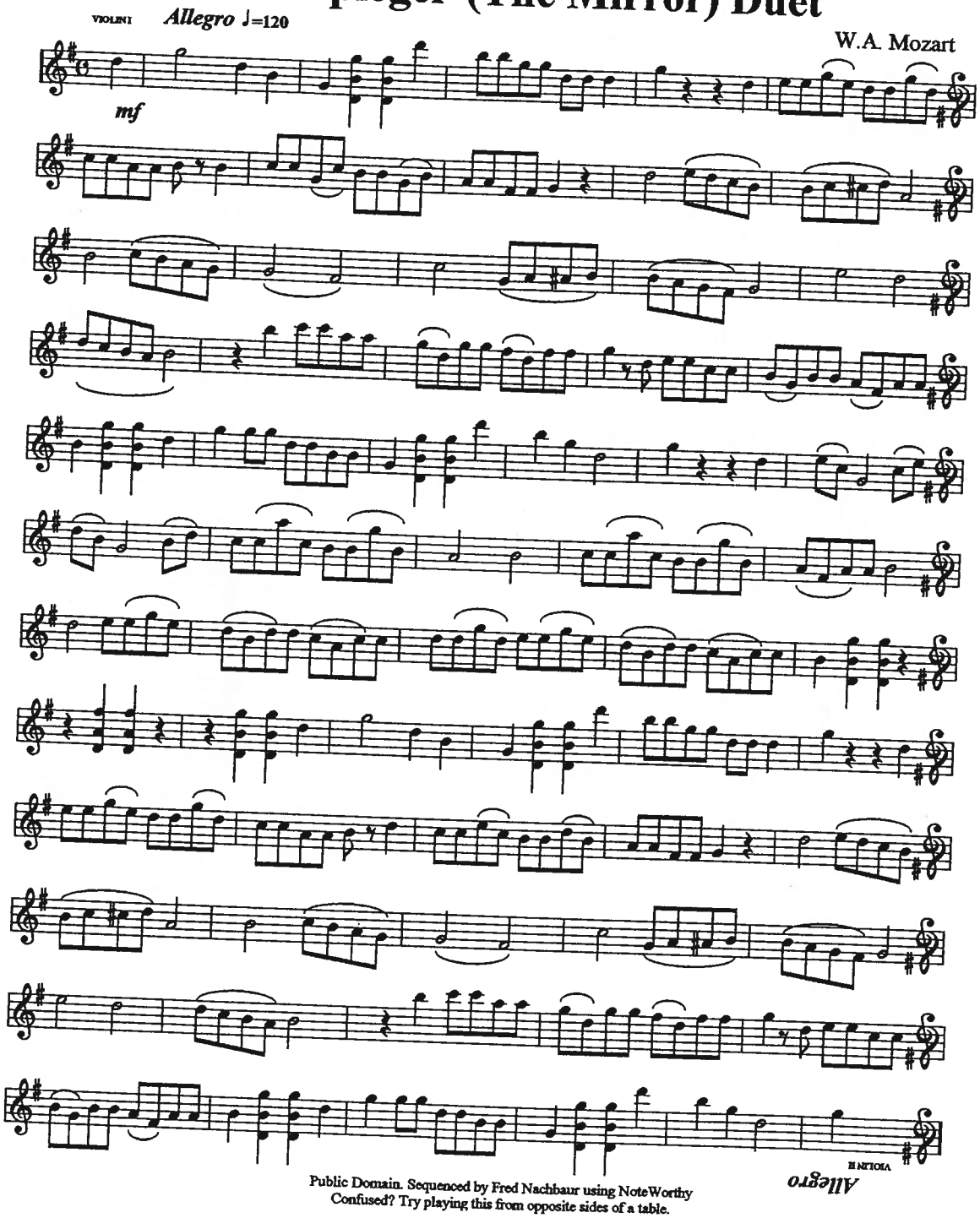
"I Got Rhythm" has an AABA structure, and a two-bar tag at the end.

We call these four equal sections A, A, B, and A. Three of the sections are the same, and one is different. Listen to the first section, and you'll be able

9b.



 by Wynton Marsalis



(Note: the attribution to Mozart is dubious)

Figure 9.6.