Composing with Mathematics: Final Projects in a Math and Music Course

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My Math and Music Course

- Two different introductory-level courses: ($<$ 20 students)
  1. *Topics in Mathematics: Math and Music* – semester long course
  2. *Math/Music: Structure and Form* (fall semester)
     Math/Music: Aesthetic Links (spring semester)
     Yearlong seminar for first year students (Montserrat program)
- Course Objectives:
  1. Investigate the multiple connections between mathematics and music. Use music to introduce interesting mathematics. Use mathematics to teach students how to "dive deep" into a topic.
  2. Develop students' skills in critical thinking and abstract reasoning.
  3. Develop a deeper appreciation for music.
  4. Integrate students' artistic and analytical skills.

Course Info

Text: *Music and Mathematics: From Pythagoras to Fractals*, edited by Fauvel, Flood and Wilson

Some Sample Topics: (second semester)

- Musical Group Theory: symmetry, group theory, symmetries of the square, examples – Bach, Haydn, Hindemith, Bartók (5 classes)
- Change (Bell) Ringing: rules of an extent, permutations, more group theory (4 classes)
- Modern Music: Schoenberg (twelve-tone method), Davies (magic squares), Xenakis, Reich (phase shift and minimalism) (6 classes)
- The Golden Section: Fibonacci numbers, nature, Mozart, Bartók (controversy!), Debussy (5 classes)
- Fractals and Chaos: iteration, the butterfly effect, Ligeti (3 classes)
- Research Paper assigned (15% of course grade) on a particular composer – part biography, part analysis of a piece of music.

Final Project

1. Compose a short (5 minutes max) piece of music incorporating some mathematical ideas discussed in the course into your work.
2. Perform piece or arrange for another person(s) to perform it.
3. Give a brief presentation before performance explaining motivations behind piece and connections to mathematics.
4. Write a brief (1 - 2 pages) paper explaining the mathematical connections in your piece and the ideas you are trying to convey.

- Performances take place at the end of the semester in a concert hall. A few class periods allocated for feedback and constructive criticism beforehand (important!)
- Students use musical software (MuseScore) to compose and format their pieces.
- 30% of final grade (in lieu of final exam).

Symmetry in Music: Musical Group Theory

How to get more music out of a little motif:

Translations (shifting graph vertically) $\Longleftrightarrow$ Transpositions (shifting notes up or down)

Vertical Reflection (symmetry between right and left) $\Longleftrightarrow$ Retrograde (music same forward and backward)

Horizontal Reflection (symmetry between top and bottom) $\Longleftrightarrow$ Inversion (what goes up, must come down). Inversions can be exact or adjusted to stay within the given key.

Adding in Rotations gives the Symmetries of the Square (D_4)

Symmetry in Music: Retrograde

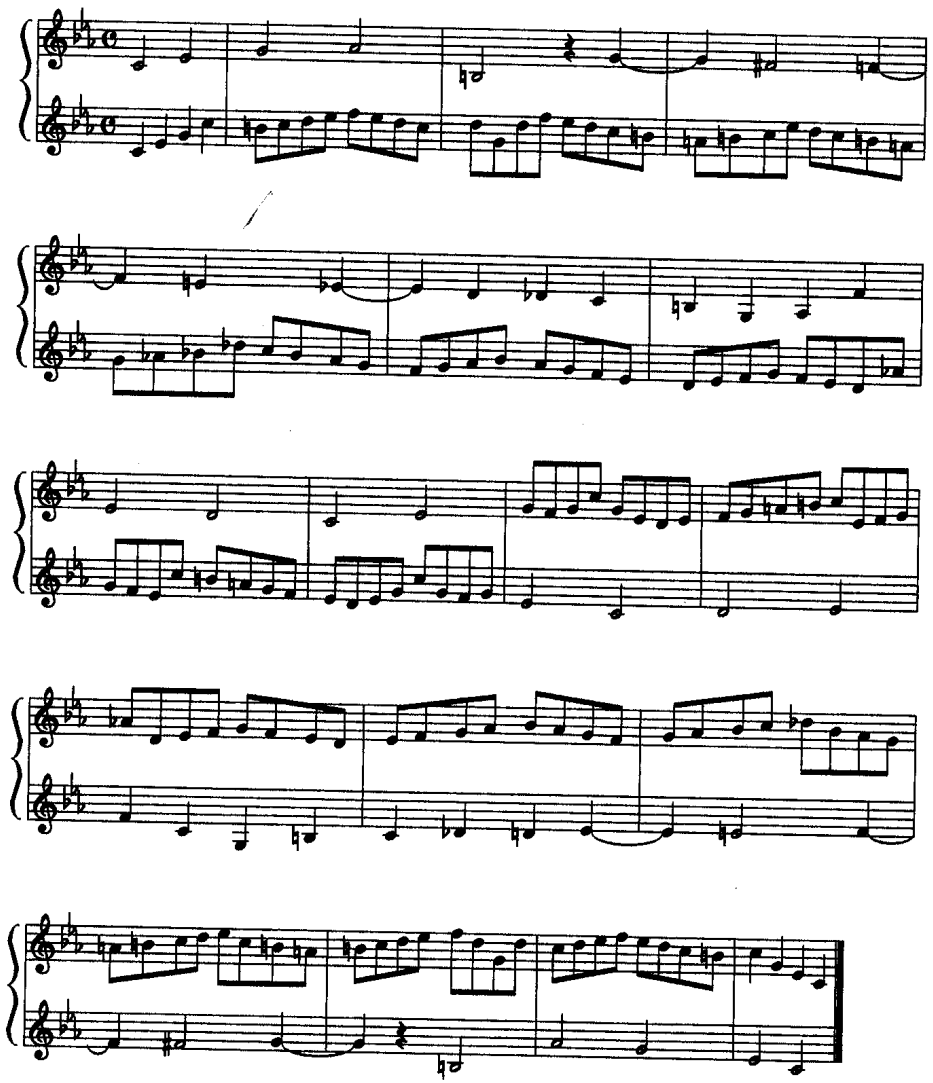


Figure: Johann Sebastian Bach, *A Musical Offering*

Symmetry in Music: Inversion

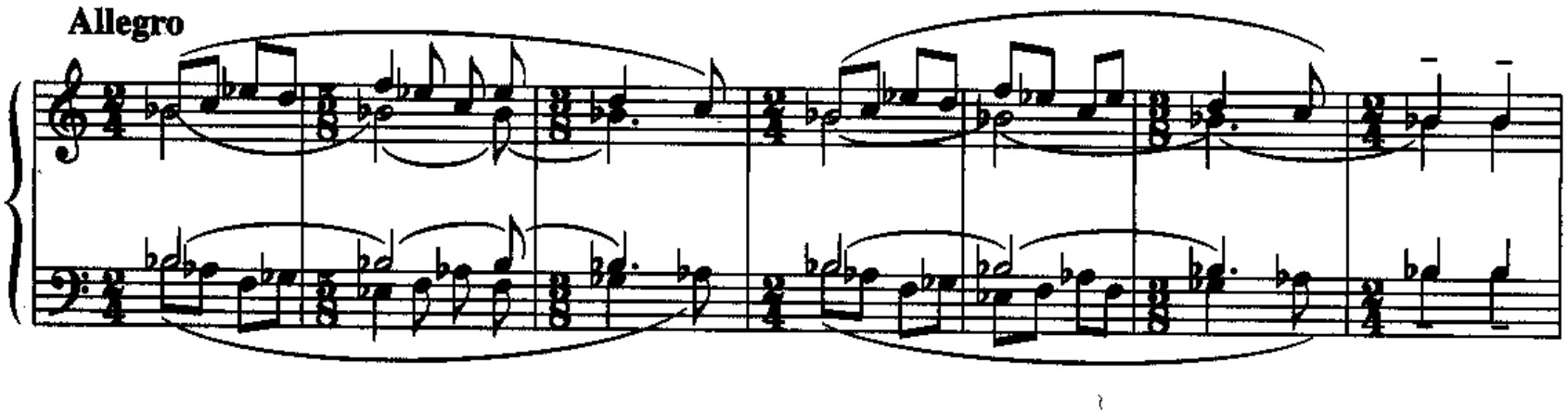


Figure: Béla Bartók, *Mikrokosmos, No. 141, Subject and reflection*

Symmetry in Music: Rotation

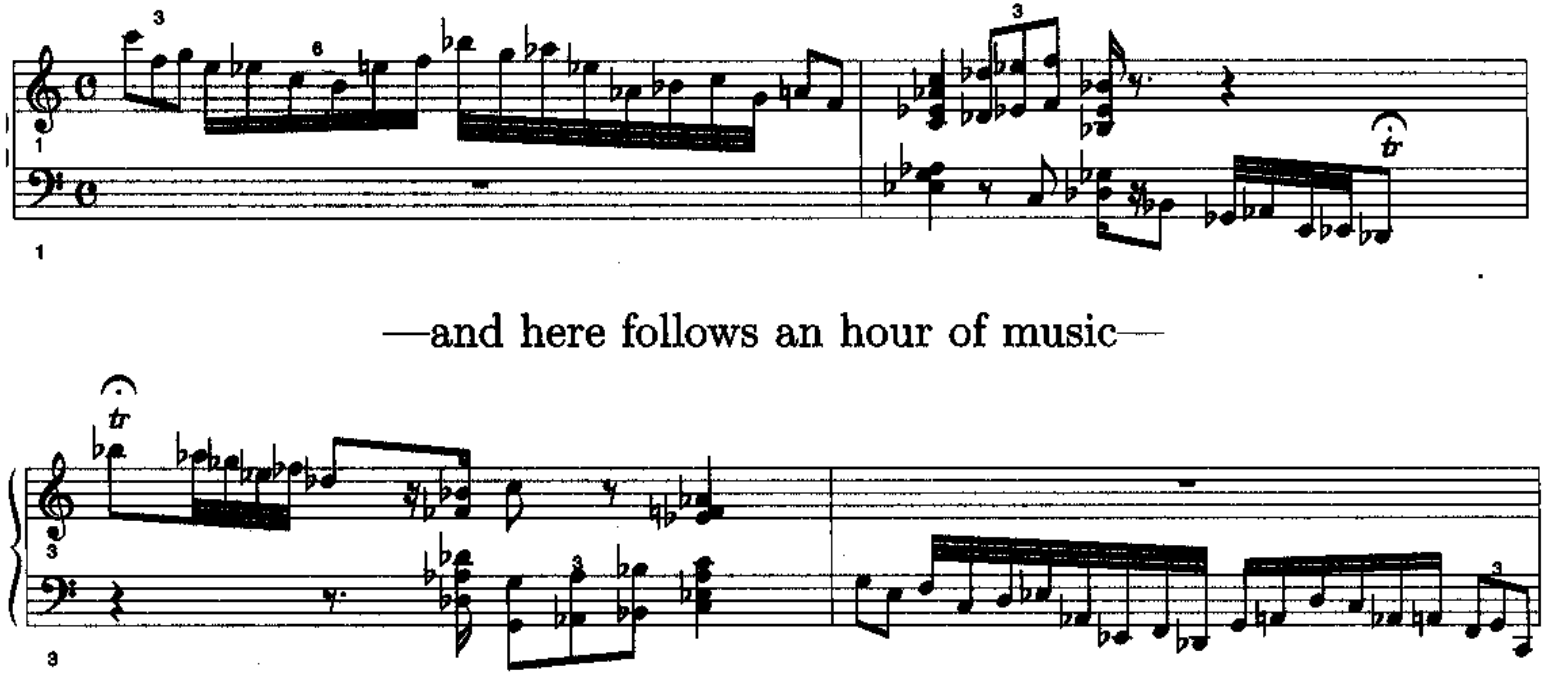


Figure: Paul Hindemith, *Ludus Tonalis* (“Tonal Game”), beginning and end. The ending Postludium is an exact retrograde-inversion of the opening Praeludium.

Some Sample Final Projects

- *Life of Pi* by Jake Miller (piano). Assigned pitches and durations to the numbers $0, 1, 2, \ldots, 9$ and then used the first 30 digits of π to generate the music. Also used symmetry to create more music.

- *Joey 12 Tone* by Joe Kramkowski (vibraphone). Wrote a tone row with 12 notes, then used symmetry operations (transpositions, retrogrades, inversions, retrograde-inversions) to construct piece. Performance on the vibraphone added an ethereal quality.

- *Ordered Pair* by Sarah DiNapoli (flute duet).

Magic Numbers by Emely Ventura (piano)

- Assigned the numbers $1, 2, \ldots, 24, 25$ to the white keys on the piano, symmetrically around middle C. Used only white keys because its easier to play.
- Inspired by compositions of Maxwell Davies using magic squares, (e.g., *A Mirror of Whitening Light*), used a 5×5 magic square to generate the music, traveling through the magic square horizontally, vertically and diagonally. "Cheated" in a few places for "harmonic reasons" (artistic liberty).

11	24	7	20	3
4	12	25	8	16
10	18	1	14	22
17	5	13	21	9
23	6	19	2	15

- Piece was exactly 65 measures long because?

  65 is the magic constant for $n = 5$

Bartalk by Christina Catalano (piano)

- Inspired by her research paper on Bartók, in particular, her study of his pieces *Mikrokosmos* and *Music for Strings, Percussion and Celesta* (MSPC), Christina used exact inversions, retrogrades and Fibonacci and Lucas numbers to construct her piece.

- Used 5 and 8 note motifs, and symmetry operations occurring at special Lucas measures, e.g., retrograde at measure 7.

- Key structural points occur at Fibonacci numbered measures, e.g., tempo change and dynamical climax at bar 55 (mimicking Bartók's 1st movement of MSPC).

- Managed to incorporate the mathematics while also writing a real piece of music. Tonality was not major/minor but not atonal either, a key feature of Bartók's music.

Star in D_5 by Allie Gitto (piano)

- "Inspired by group theory and the dihedral group," Allie first computes the group multiplication table for D_5. Then, she creates a correspondence between the first five natural numbers and five notes ($1 \leftrightarrow$ A, $2 \leftrightarrow$ B, $3 \leftrightarrow$ C, $4 \leftrightarrow$ D, $5 \leftrightarrow$ F), and uses the group elements and multiplication table to construct her piece.

- Work written in $\frac{5}{8}$ so that each group element fits precisely in one measure.

- Piece opens with each element played once, announcing the group. Then, a fixed element g is chosen (e.g., $g = R_{72}$) to be played in the right hand while the left hand cycles through all elements except for e and g^{-1}. The composition of the right- and left-hand elements (essentially a left coset) are played in unison. In essence, the group multiplication table is being portrayed musically!

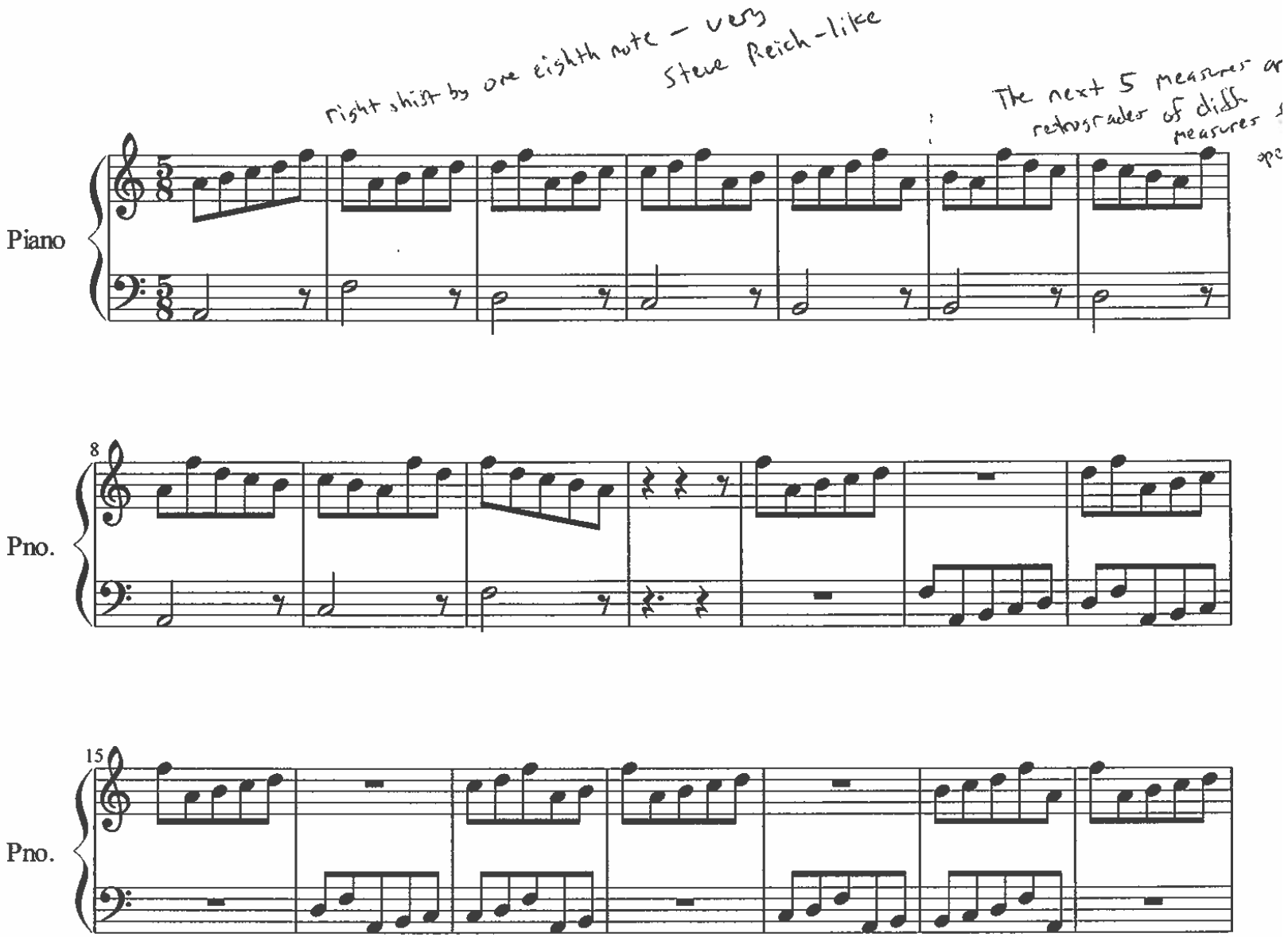
Star in D_5 (cont.)

- Piece concludes with each rotation composed with its inverse to obtain the identity in both hands (a recognizable ascending scale). This is exactly the procedure used in bell ringing, where a consecutive sequence of numbered bells is played (called "rounds") to announce the start and finish of a piece.

Nice job!

	e	R_{72}	R_{144}	R_{216}	R_{288}	P_4	P_5	P_1	P_2	P_3
e	e	R_{72}	R_{144}	R_{216}	R_{288}	P_4	P_5	P_1	P_2	P_3
R_{72}	R_{72}	R_{144}	R_{216}	R_{288}	e	P_2	P_3	P_4	P_5	P_1
R_{144}	R_{144}	R_{216}	R_{288}	e	R_{72}	P_5	P_1	P_2	P_3	P_4
R_{216}	R_{216}	R_{288}	e	R_{72}	R_{144}	P_3	P_4	P_5	P_1	P_2
R_{288}	R_{288}	e	R_{72}	R_{144}	R_{216}	P_1	P_2	P_3	P_4	P_5
P_4	P_4	P_1	P_3	P_5	P_2	e	R_{216}	R_{72}	R_{288}	R_{144}
P_5	P_5	P_2	P_4	P_1	P_3	R_{144}	e	R_{216}	R_{72}	R_{288}
P_1	P_1	P_3	P_5	P_2	P_4	R_{288}	R_{144}	e	R_{216}	R_{72}
P_2	P_2	P_4	P_1	P_3	P_5	R_{72}	R_{288}	R_{144}	e	R_{216}
P_3	P_3	P_5	P_2	P_4	P_1	R_{216}	R_{72}	R_{288}	R_{144}	e

Star in D5


right shift by one eighth note – very Steve Reich-like
The next 5 measures ar
retrograde of diff
measures
Piano
Pno.
8
Pno.
15

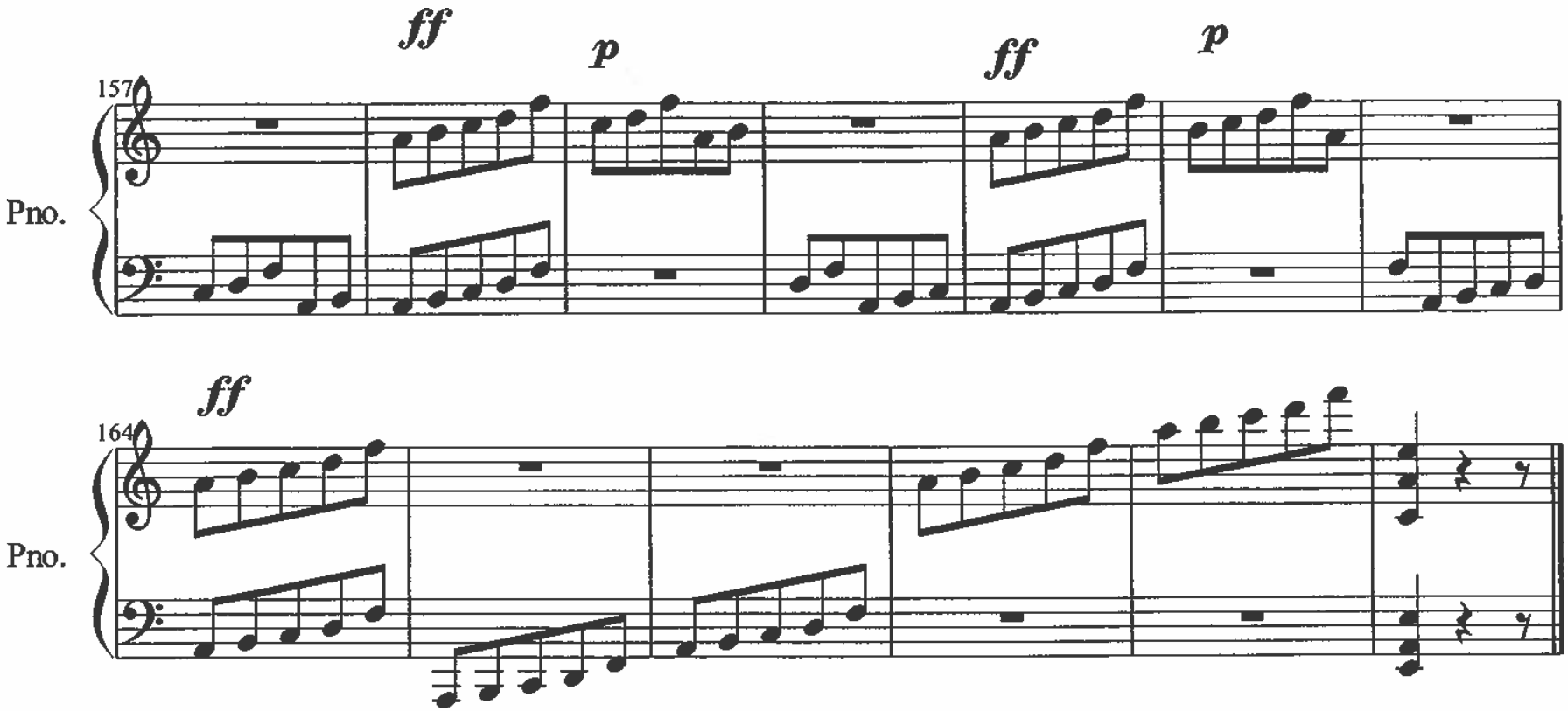


Figure: The conclusion of *Star in* D_5.

Newcomer's Waltz by Megan Whitacre (piano)

Figure: Megan Whitacre, star of the HC Ballroom Dance Team.

Features of *Newcomer's Waltz*

- Artistic vision inspired by her experiences as a dancer and intended to reflect the challenges and inner struggles of being a dancer as well as a first-year college student. As a result, the music is "a little eerie," employing a minor scale with a sharp 4th scale degree and emphasizing tritones and minor seconds.

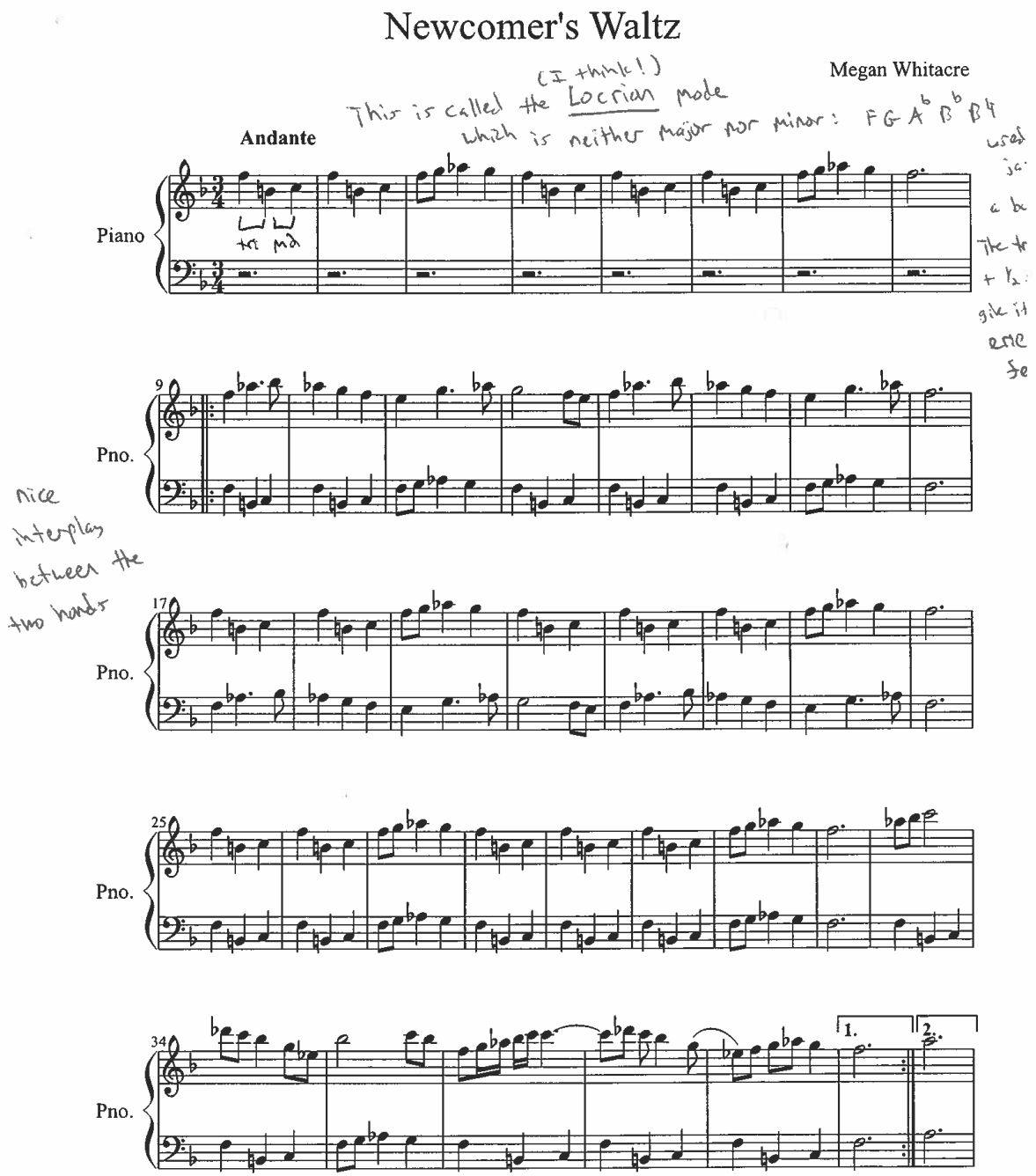
- Piece played on piano (off to side by friend) while Megan and her dance partner performed a routine choreographed to match the music and the mathematics. Stunning!

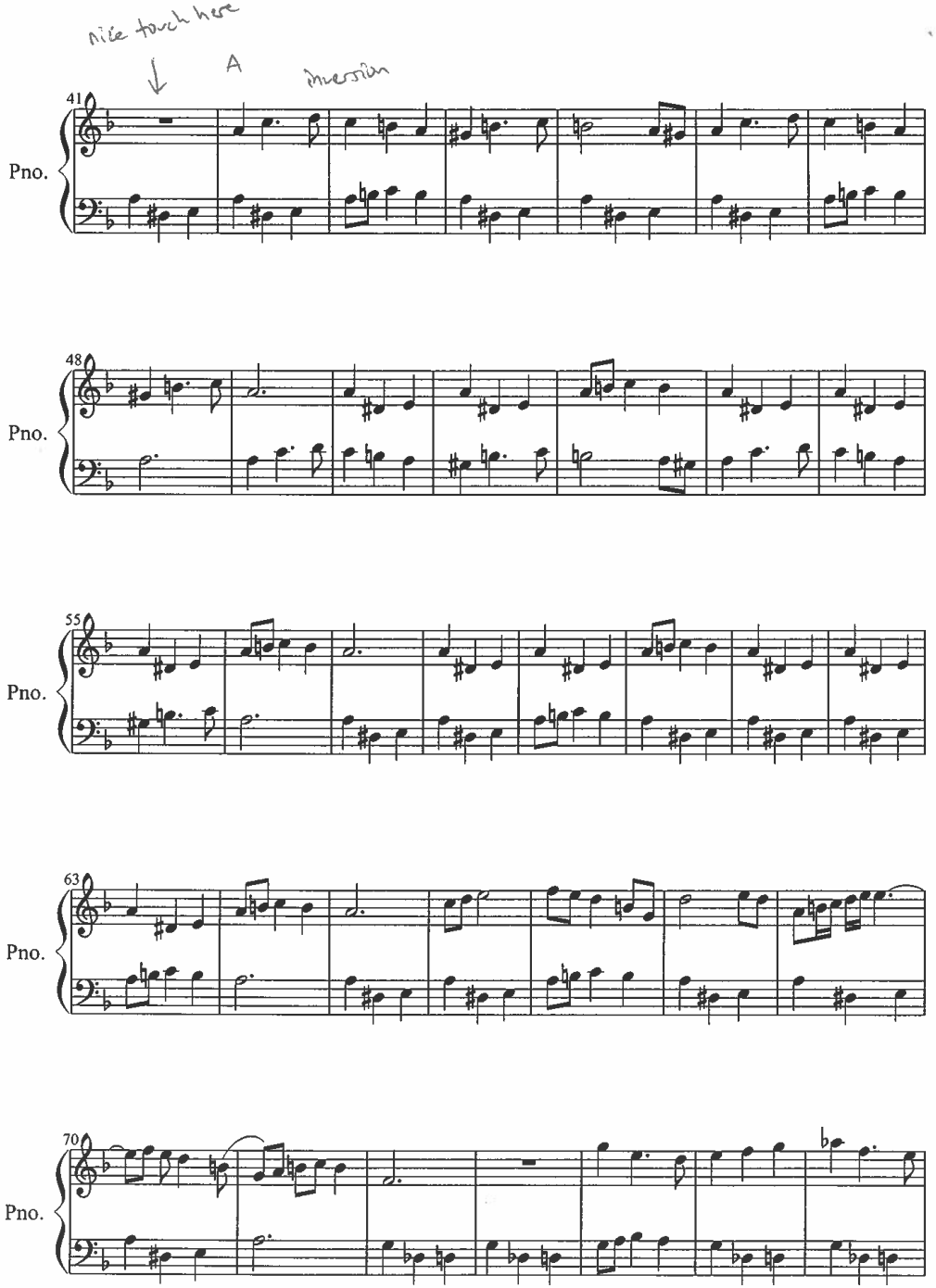
- Inspired by Bach, a haunting opening melody in the right-hand is transposed and inverted multiple times. A counter-subject enters in the right hand in measure 9 while the melody shifts to the left hand.

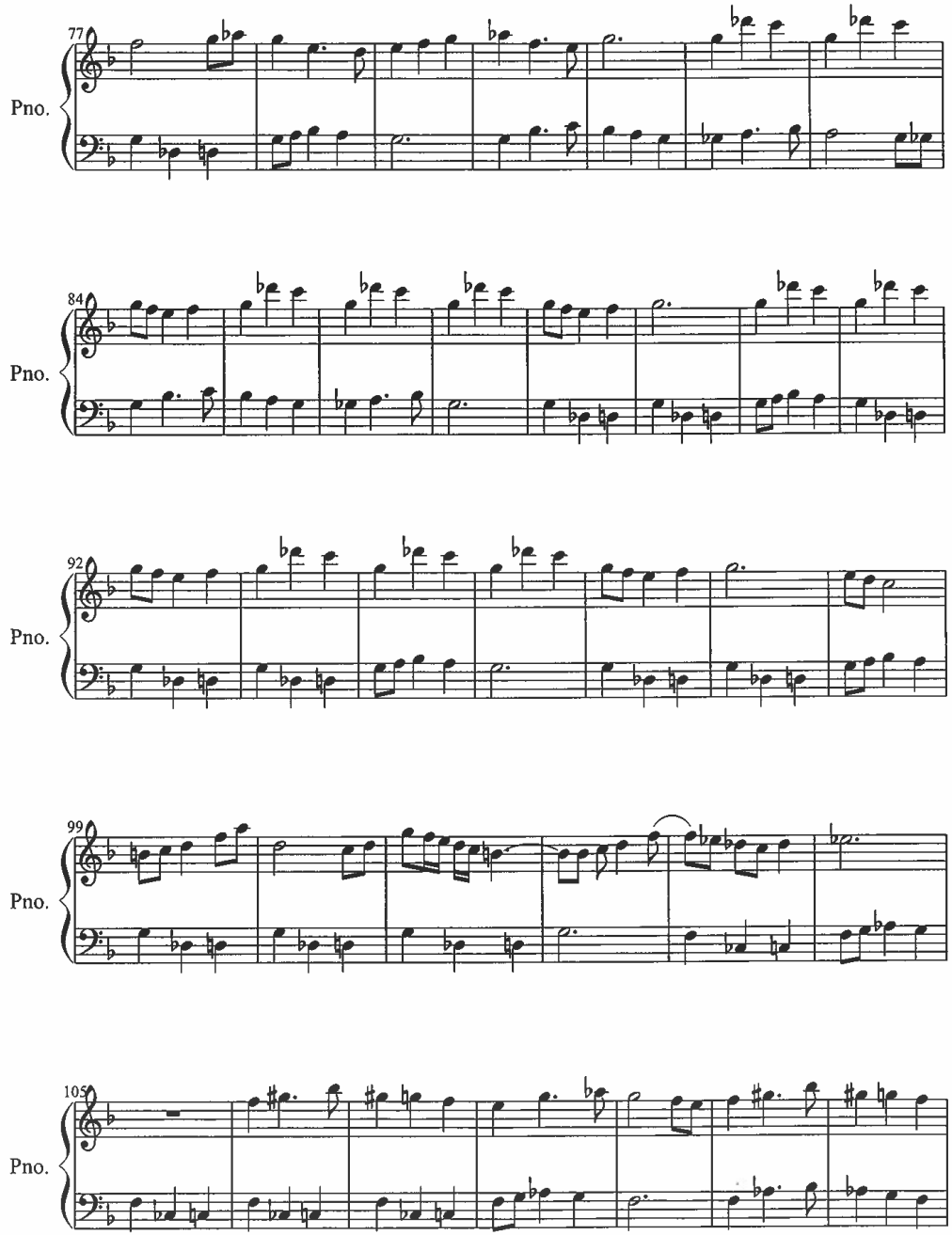
More Features of *Newcomer's Waltz*

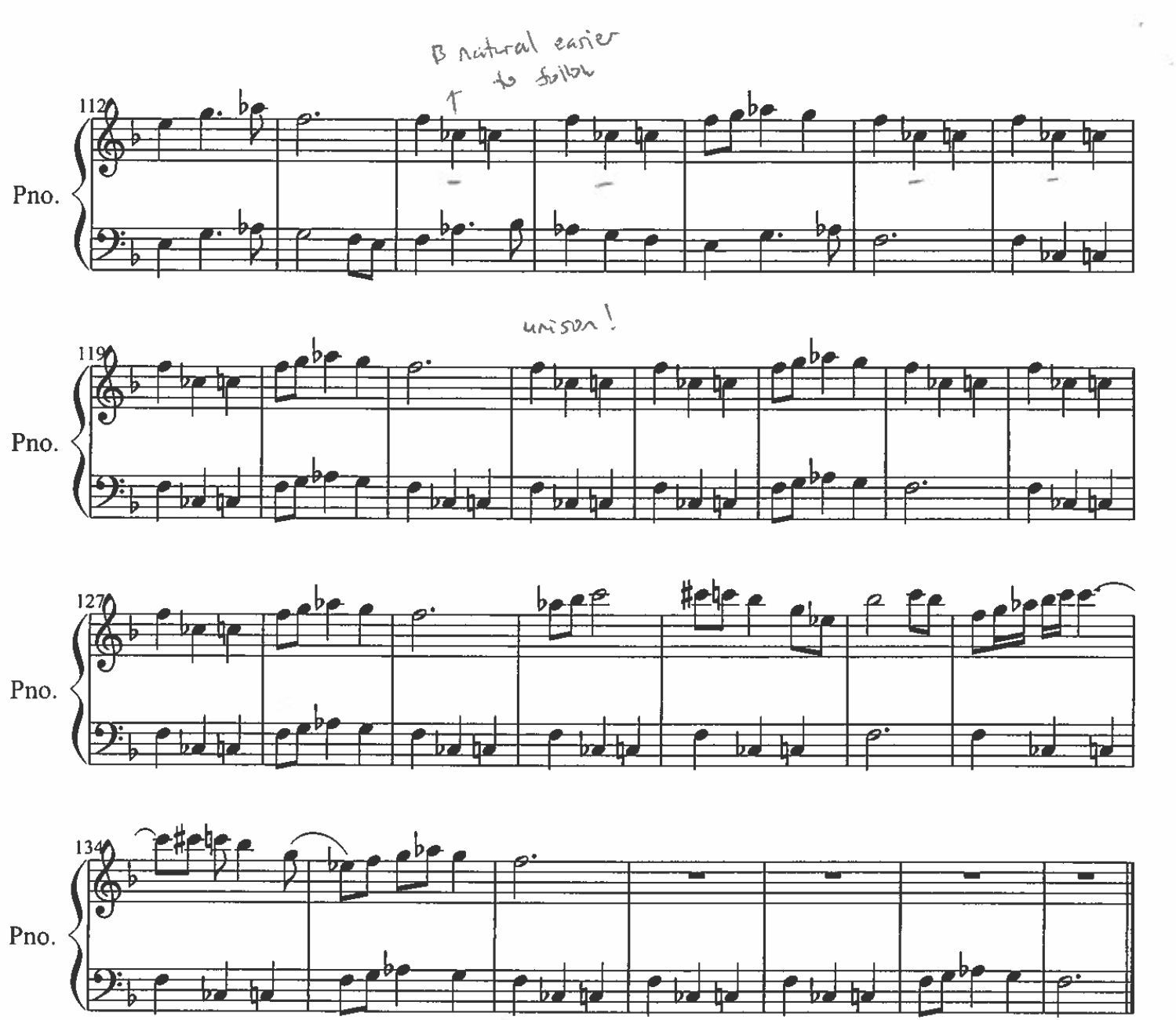
- The choreography of the dance illustrates the mathematical concepts present in the work. When the two hands are in unison, so are the dancers. When the melody is inverted, the dancers face each other to "reflect" one and other's movements. When a phase shift happens in the music, a time delay takes place with one dancer 3 beats behind the other.

- Borrowing a practiced technique of Steve Reich's, a simple phase shift is used in the right hand melody (inserting one measure rests at bars 41, 73 and 105) to create different juxtapositions of the melody and counter-subject. "This phase shift changes the way the entire piece comes together."


Newcomer's Waltz
Megan Whitacre
(I think!)
This is called the Locrian mode
which is neither major nor minor: F G A♭ B♭ B♮
Andante
Piano
tri mid
used
ja
a be
the tr
+ ½
give it
ene
fe
9
Pno.
nice
interplay
between the
two hands
17
Pno.
25
Pno.
34
Pno.
1.
2.


Nice touch here
A
inversion
41
Pno.
48
Pno.
55
Pno.
63
Pno.
70
Pno.


77
Pno.
84
Pno.
92
Pno.
99
Pno.
105
Pno.


B natural easier to solfa
unison!
112
119
127
134
Pno.
Pno.
Pno.
Pno.