Was "Pythagoras" a Babylonian?

Amherst College Mathematics Colloquium
September 29, 2011

John Little
little@mathcs.holycross.edu

Plan for this talk

- The "Pythagorean" theorem through history.
- Mathematics of the Old Babylonian period and different interpretations of some key cuneiform tablets: YBC 6967, YBC 7289, Plimpton 322
- Should we *care* who found these ideas *first* and whether they get proper credit?

Who was the historical Pythagoras?

- Greek mathematician, philosopher, religious leader/mystic from Samos
- Ca. 580 – 500 BCE
- Very little is known about his life or mathematical work
- According to T. L. Heath (famous editor of Euclid's *Elements*) the "Pythagorean Theorem" *was* traditionally ascribed to Pythagoras, but the surviving sources (Plutarch, Cicero, Proclus, …) all come much after his time

Proclus (5^{th} century CE) on the theorem

"If we listen to those who wish to recount ancient history, we may find some of them referring this theorem to Pythagoras, and saying that he sacrificed an ox in honor of his discovery. But for my part, while I admire those who first observed the proof this theorem, I marvel more at the writer of the *Elements,* [who] made it fast by a most lucid demonstration."

Some interesting tidbits

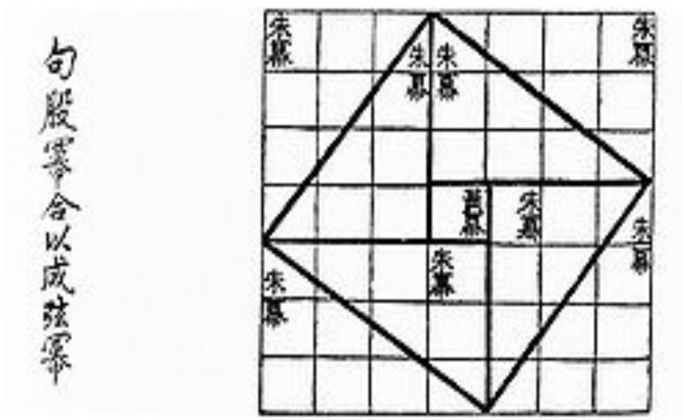
- A Greek tradition – Pythagoras was said to have traveled widely to Egypt, Babylon, and possibly even as far as India
- In general, the Greeks were more than willing to acknowledge their indebtedness to Egypt and Mesopotamia for the basis of much of what they took and developed

India -- The *Sulbasutras*

- A collection of mathematical texts that can be traced back to the 8^{th} century BCE – connected with construction of altars for Vedic religion
- Use integer Pythagorean triples such as (3,4,5), (5,12,13), (8,15,17), (12,35,37)
- Give a statement of the general theorem, and
- A proof in a special case

China -- The *Zhou Bi Suan Jing*

- Chinese “Arithmetical Classic of the Gnomon”
- Earliest surviving copies about 100 BCE, but thought to date from much earlier (perhaps 1100 BCE)
- Contains a chapter on the ``*go-gou*” theorem



Could it be even older?

- Mesopotamia, the ``land between the rivers'' – Tigris and Euphrates (currently divided between Iraq, Iran, Syria).

A *very* long history

- ca. 5500 BCE -- First village settlements in the South
- ca. 3500 - 2800 BCE -- Sumerian city-state period, first *cities*, first pictographic texts
- ca. 3300 - 3100 BCE -- first *cuneiform* writing
- created with a reed stylus on a wet clay tablet, then sometimes baked
- In a dry climate, these records are *very* durable!

Cuneiform writing

- Different combinations of up-down and sideways wedges were used to represent *syllables*
- Was used to represent many different spoken languages over a *long* period – 1000 years +
- We'll see the way *numbers* were represented in this system shortly.
- Used for everything – bureaucratic records, literature, mathematics, ...

A tablet with cuneiform writing

- Note the limited collection of forms you can make with a wedge-shaped stylus:

Concentrate on southern area

ca. 2800 - 2320 BCE -- Early Dynastic Period, Old Sumerian literature

ca. 2320 - 2180 BCE -- Akkadian (Sumerian) empire, first real centralized government

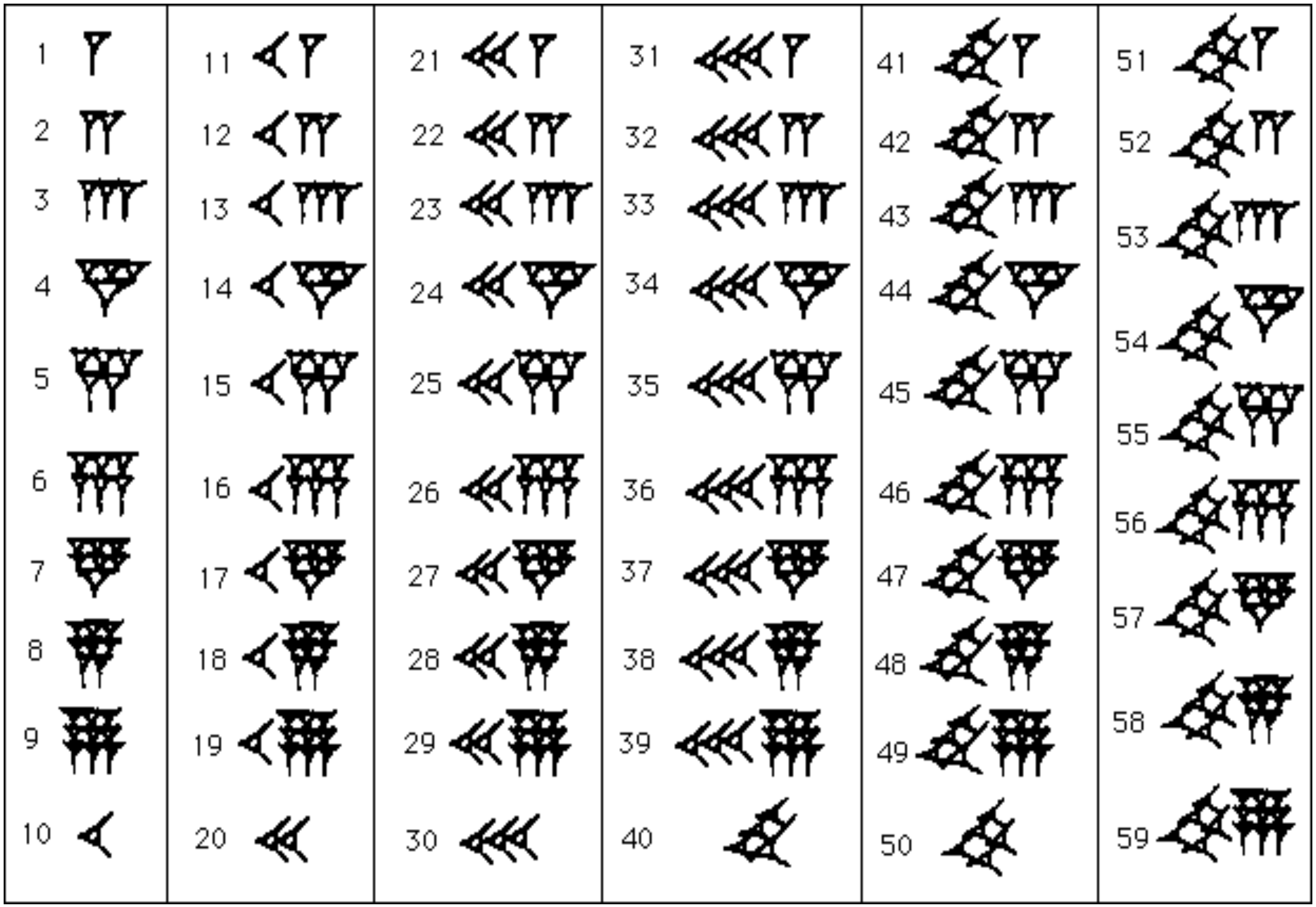
ca. 2000 BCE -- collapse of remnant of Sumerian empire

ca. 2000 - 1600 BCE -- Ammorite kingdom -- "Old Babylonian Period"-- Hammurabi Code, mathematics texts, editing of Sumerian *Epic of Gilgamesh* (~ contemporaneous with Egyptian “Middle Kingdom” and *Ahmes* and *Rhind* papyri.)

Babylonian *mathematics*

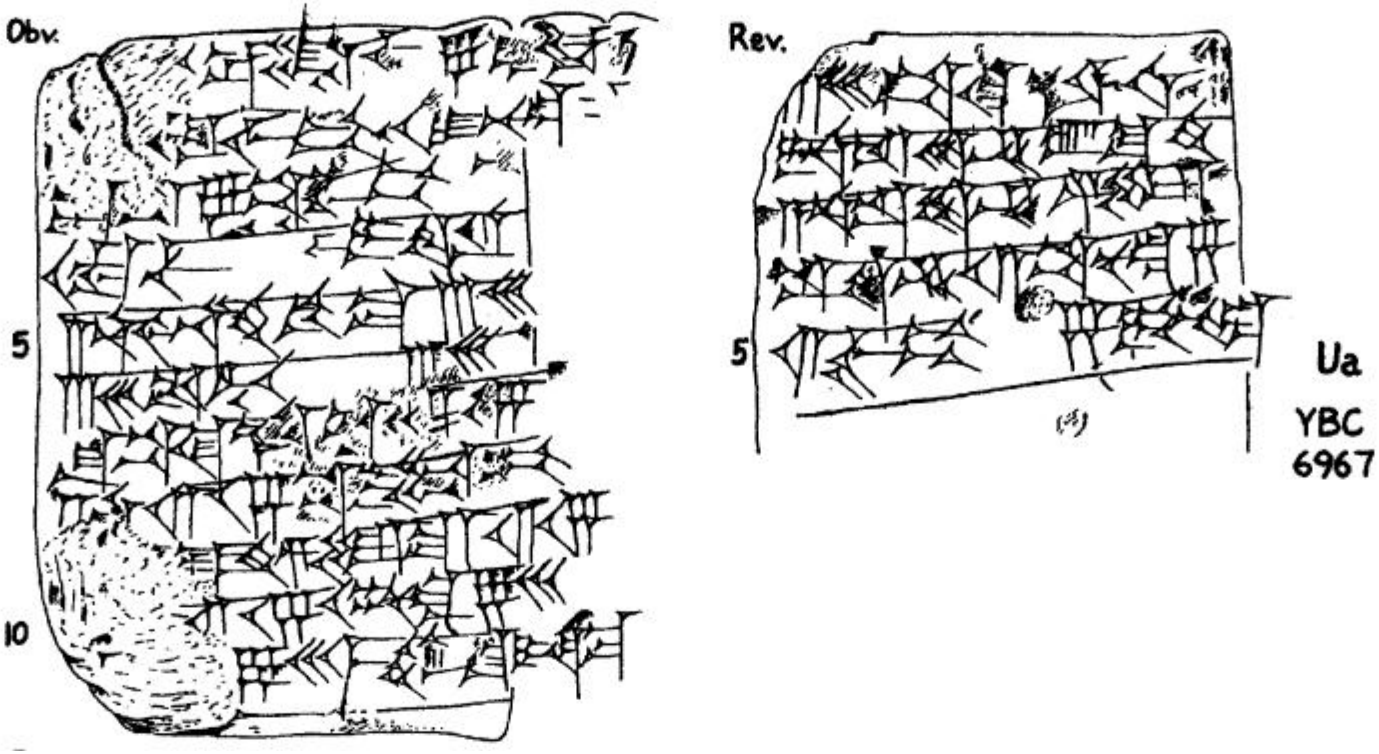
- Taught in *schools* for scribes connected with governmental and religious centers
- Used a distinctive *base-60 positional number system,* including base-60 fractions (but no symbol for zero, so sometimes ambiguous!)
- Featured very extensive and sophisticated calculations
- No evidence of a concept of general *proof* – everything based on examples or models leading to general methods.

The number system



A famous mathematical text

- The tablet known as ``YBC 6967" (Note: YBC = “Yale Babylonian Collection”)



YBC 6967

- Recognized as a mathematical text and translated by O. Neugebauer and A. Sachs (1945).
- Essentially a *mathematical problem* (probably set to scribal students in the city of Larsa) and a step-by-step *model solution*.
- **The problem**: A number x exceeds $60/x$ by 7. What are x and $60/x$?
- **Comment**: Many similar tablets with variants of this problem have also been recognized.

The Babylonian solution

- Paraphrase of Neugebauer and Sachs's translation of the solution given on YBC 6967 (with decimal fractions!)
- *Halve* the 7 to get 3.5
- *Square* the 3.5 to get 12.25
- Add the 12.25 to 60 to get 72.25, and extract *square root* to get 8.5
- Subtract the 3.5 from 8.5 to get 5, which is $60/x$
- Then $x = 12$

What's going on here?

- Possible explanation: The original problem asks for a solution of $x = 60/x + 7$, or
- $x^2 - 7x - 60 = 0.$
- Larger solution of $x^2 - px - q = 0,\ p,q > 0$ can be written as: $x = p/2 + \sqrt{(p/2)^2 + q}$
- *This is exactly what the "recipe" given in the YBC 6967 solution does(!)*

Perils of doing mathematical history

- Does that mean that the Babylonians who created this problem text knew the *quadratic formula*?
- Best answer to that one: While they doubtless could have *understood it*, from what we know, they just did not think in terms of general formulas that way. So, ***probably no***, not really.
- *Conceptual anachronism* is the (“amateur” or professional) mathematical historian's worst temptation.

So what *were* they doing?

- Neugebauer's answer – it could have been ``quadratic algebra'' based on the identity
  (*) $((a+b)/2)^2 - ((a-b)/2)^2 = a\,b$
- Letting $a = x$, $b = 60/x$, then $a - b = 7$ and $a\,b = 60$ are known from the given information.
- The steps in the YBC 6967 solution also correspond exactly to one way to solve for a and b from (*)
- But isn't this *also* possibly *anachronistic??*

More recent interpretations

- More recent work on Babylonian problem texts including YBC 6967 by Jens Hoyrup and Eleanor Robson has taken as its starting point the “geometric flavor” of the *actual language* used in the solution:
- Not just “halve the 7” but “break the 7 in two”
- Not just “add” the 3.5^2 to the 60, but “append it to the surface”
- Not just “subtract” 3.5, but “tear it out.”

YBC 6967 as “cut and paste”

- So in fact Hoyrup proposed that the solution method given on YBC 6967 could be visualized as “cut and paste” geometry like this – [do on board].
- The (subtle?) point: this is *mathematically equivalent* to Neugebauer's algebraic identity (*), of course.   But Hoyrup argues that it seems to “fit” the linguistic evidence from the text and what we know about the cultural context of Babylonian mathematics better.

Babylonian *geometry*(?)

- More importantly, it serves to plant the idea that (contrary to what Neugebauer thought and wrote many times), Babylonian mathematics contained more than a little *geometric* thinking as well as *algebraic ideas*.
- Finally, note how close we are here to Pythagorean triples, and the Chinese *go-gou* theorem – at least with our knowlege(!)

Was “Pythagoras” Babylonian?

- (Had it ever occurred to you that the quadratic formula and the Pythagorean theorem might be this closely related? It certainly never had to me before I started looking at this history(!))
- What can we say about whether the Babylonians really understood a general Pythagorean Theorem? There are many tantalizing hints, but nothing definitive (a difference between mathematics itself and its history!)

A First Piece of Evidence

- The tablet YBC 7289

How did they do it?

- Short, frustrating answer – as with so many other things, ***we don't know***.
- However, a more common approximation of $\sqrt{2}$ they used: $\sqrt{2} \doteq 17/12 \doteq 1.416666$ can be obtained starting from $x = 1.0$ by two iterations of "Newton's Method" on $x^2 - 2 = 0$.
- Whoever created this tablet may have done more extensive computations of a related sort (but not *exactly* that).

YBC 7289

- The numbers here are:
- On one side 30 – evidently to be interpreted the fraction 30/60 = ½
- The top number written on the diagonal of the square are: 1.24:51:10 – in base 60, this gives approximately 1.414212963...
- Note: $\sqrt{2} \doteq 1.414213562...$
- The lower one is .42:25:35 – exactly half of the other one.

More evidence?

- There is another very well-known tablet known as “*Plimpton 322*" that gives more evidence of the degree to which the Babylonians appreciated the general $a\uparrow 2 + b\uparrow 2 = c\uparrow 2$ relation and integer Pythagorean triples

Plimpton 322

- The most famous (and *enigmatic!)* Old Babylonian mathematical text

Interpreting Plimpton 322

- The contents of this tablet form essentially a table of integer Pythagorean triples. For instance, row 11 of the table includes the numbers $c = 75$, $a = 45$ from the nonprimitive triple (45,60,75) = 15 (3,4,5).
- Neugebauer interpreted this table as a systematic application of the generating formulas related to (*) from before:

$$a = p^2 - q^2 \qquad b = 2\,p\,q \qquad c = p^2 + q^2$$

Interpreting Plimpton 322, continued

- The first (leftmost) column contains the values of $(c/b)^2$ for the corresponding triangle. So was this the first *trigonometry table??* (Discussed by R. C. Buck in a *Monthly* article from 1980.)
- *Doubtful – almost certainly anachronistic(!)*
- Evidence that the Babylonians appreciated the general Pythagorean relationship, although (according to their style, they probably did not think about trying to find a general *proof*) -- ??

Interpreting Plimpton 322, continued

- Most recently, Eleanor Robson has argued that this table was a record of particular numbers for instances of the problem genre of YBC 6967, based on connection with "reciprocal pairs" $x, 60/x$ (or $1/x$) and what we would write as $(x + 1/x)^2 = 4 + (x - 1/x)^2$ It would have been something like the teacher's notes(!)
- When you see the actual artifact, it clearly originally contained additional columns of information on the left(!)

Interpreting Plimpton 322, concluded

- Until and unless the missing portion shows up in a museum drawer somewhere (and we can hope – this sort of thing is hardly unknown in archaeology!) we may never know the "last word" on Plimpton 322.
- If nothing else, this tablet is the record of some truly heroic hand calculations in a *nontrivial* number representation – typical of the Pythagorean triples included:

(4601,4800,6649)

Should we care who did what when?

- Consider the following opinions about the history of mathematics:
- “Compared with the accomplishments of … the Greeks, the mathematics of the Babylonians is the scrawling of children just learning to write as opposed to great literature." – *Morris Kline, Mathematics for the Nonmathematician*
- “ … what the Greeks created differs as much from what they took over from … the Babylonians as gold differs from tin.” – *ibid.*

More in the same vein

- “[The Greeks] are not clever school boys or scholarship candidates, but `fellows of another college.' ” – J. E. Littlewood
- “Hindu” mathematics “… was a mixture of pearl shells and sour dates … of costly crystal and common pebbles.” -- al Biruni, 11th century CE (a Muslim writer, on earlier Indian work)
- Doesn't it seem troubling how some writers go out of their way to *denigrate* earlier cultures?

Conclusion

- Mathematical thinking is one of the most universal human activities – giving historical credit where it is due can only increase our understanding and appreciation for the way all of our forebears contributed to our current knowledge.
- Although the Greeks themselves *always* freely acknowledged their indebtedness to Egypt and Babylonia, the *degree to which* their work built on earlier work and the paths by which those ideas were transmitted are still not that well understood.

Further Reading

- G.G. Joseph, *The Crest of the Peacock,* Princeton University Press
- The *Mathematics* of *Egypt*, *Mesopotamia, China, India, and Islam*: A Sourcebook, Katz, V.J., ed. Princeton University Press
- E. Robson, *Neither Sherlock Holmes Nor Babylon*, A Reassessment of Plimpton 322, *Historia Mathematica,* **28** (2001), 167-206.