Mathematics 375 – Probability and Statistics 1
Problem Set 7 – *due*: Wednesday, November 5, 2003

From the text: Chapter 4/69, 70, 71, 72, 75, 83, 87, 91, 97, 98, 101.

Additional Problems

In most cases, the *only* way to compute probabilities with Gamma- and Beta-distributed random variables is to use numerical methods on the appropriate integrals of the pdf's. However, in some special cases, there are interesting connections with some of the discrete random variables we have studied before.

A) In this problem you will study a relation between continuous Gamma distributions and discrete Poisson distributions in the case that the parameter α of the Gamma distribution is a positive integer and $\beta = 1$.

1) Let α be a positive integer and consider a Gamma-distributed random variable Y with parameters α positive $\in \mathbf{Z}$, and $\beta = 1$. Let $\lambda > 0$ be arbitrary. Using integration by parts and a proof by mathematical induction, show that

$$\begin{aligned} P(Y \geq \lambda) &= \frac{1}{\Gamma(\alpha)} \int_{\lambda}^{\infty} y^{\alpha-1} e^{-y}\, dy \\ &= \sum_{n=0}^{\alpha-1} \frac{\lambda^n}{n!} e^{-\lambda} \end{aligned}$$

(Note that the last line here is a sum of probabilities for a Poisson discrete random variable(!)).

2) Suppose Y has a Gamma distribution with $\alpha = 3$, $\beta = 1$. Using the result of part 1 and the tables in our text, find $P(Y \geq 5)$.

B) In this problem you will study a relation between continuous Beta distributions and discrete binomial distributions in the case that the parameters α, β of the Beta distribution are positive integers.

1) Let Y be a continuous random variable with a Beta distribution where α and β are positive integers. Let $0 < y < 1$. Show that

$$\begin{aligned} P(0 \leq Y \leq p) &= \frac{1}{B(\alpha,\beta)} \int_0^p y^{\alpha-1}(1-y)^{\beta-1}\, dy \\ &= \sum_{n=\alpha}^{\alpha+\beta-1} \binom{\alpha+\beta-1}{n} p^n (1-p)^{\alpha+\beta-1-n} \end{aligned}$$

(Note that this is a sum of binomial probabilities(!)).

2) Suppose that Y has a Beta distribution with $\alpha = 4$ and $\beta = 7$. Use the result of part 1 and the tables in the text to find $P(0 \leq Y \leq .3)$.