Returning to the Roots of Mathematics: A Personal Journey

Faculty Scholarship Lunch

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Who says mathematics doesn't get contentious?

From an extraordinary "flame war" carried out mid-1970's in *Arch. for Hist. of Exact Sci.*:

" ... history of mathematics has been typically written by mathematicians ... who have either reached retirement age and ceased to be productive in their own specialties or become otherwise professionally sterile. ... The reader may judge for himself how wise a decision it is for a professional to start writing the history of his discipline when his only calling lies in professional senility."

I probably agreed when this was pointed out to me by Tony Stankus in the early 1980's(!) Wonderful irony and/or a complete confirmation of the author's point: I've recently gotten interested in issues this article dealt with(!) More on that later ...

You can probably sympathize

- That awkward question ... “So, what do you do?”
- Because I’m an *algebraic geometer* by training, often start out by saying, “Well, I’m a mathematician – I study geometric objects defined by certain kinds of algebraic equations, ... ”
- “For instance, the *conic sections* (ellipses, parabolas, hyperbolas) all defined by second degree polynomial equations in two variables:

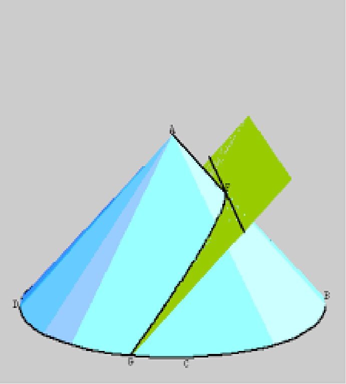
  $$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

  for some coefficients A, B, C, D, E, F.”
- “It all goes back to the ancient Greeks ... ”

This talk

- *Exactly how* ancient Greek mathematicians such as Apollonius really think about the conic sections?
- Return to the roots! Goal: Read Apollonius (and Descartes) but not filtered through modern "interpretations," "explanations" and translations
- To do that, I had to learn some ancient Greek ... lucky I'm at Holy Cross with our world-class and wonderfully welcoming Classics department!
- Want to share some first observations from this new aspect of my scholarship

Context – previous work on conics



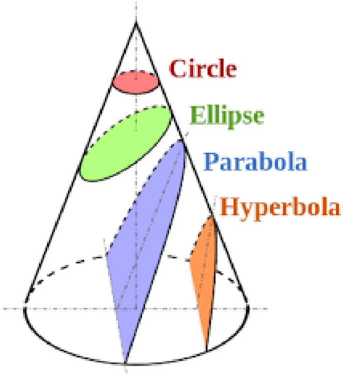
(Image by H. Mendel, Cal. State LA)

- Menaechmus (ca. 380 - 320 BCE; in Plato's circle) first, developed by Aristaeus (pre-Euclid) and Euclid (ca. 300 BCE)
- We think they got the different conic sections from different cones: the parabola as a "section of a right-angled cone"

Apollonius of Perga, ca. 262–190 BCE

- Active roughly 75 - 100 years after time of Euclid (ca. 300 BCE); slightly younger than Archimedes (287–212 BCE); studied in Alexandria
- The *Conics*: Books I, II, III, IV – "Elements of conics" – survive in Greek versions.
- Books V, VI, VII – "Researches on conics" – only known in Arabic
- Book VIII – ? (lost – several attempts at "reconstruction" including one by E. Halley, 1710 CE)

Apollonius's framework



(Image from Wikipedia, "Conic Sections")

- Conic surfaces generated by lines through a vertex and points on a circle
- All conics are obtained by slicing any one conic surface by different planes.
- Get plane curves, but the construction inherently uses geometry in 3 dimensions(!)

A sample: Apollonius, Book I Proposition 11 – definition of the parabola

If a cone is cut by a plane through its axis, and also cut by another plane cutting the base of the cone in a straight line perpendicular to the base of the axial triangle, and if, further, the diameter of the section is parallel to one side of the axial triangle, and if any straight line is drawn from the section of the cone to its diameter such that this straight line is parallel to the common section of the cutting plane and of the cone's base, then this straight line to the diameter will equal in square the rectangle contained by (a) the straight line from the section's vertex to where the straight line to the diameter cuts it off and (b) another straight line which has the same ratio to the straight line between the angle of the cone and the vertex of the section as the square on the base of the axial triangle has to the rectangle contained by the remaining two sides of the triangle. And let such a section be called a parabola. (R.C.Taliaferro's translation)

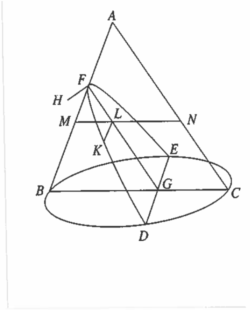
In case that wasn't clear ...

- J. Kepler: (in reference to one of his own works) "If anyone thinks that the obscurity of this presentation arises from the perplexity of my mind, ... I urge any such person to read the *Conics* of Apollonius. He will see that there are some matters which no mind, however gifted, can present in such a way as to be understood in a cursory reading. There is need of meditation, and a close thinking through of what is said."
- Descartes, from *La Géométrie*: " *... je vous prie de remarquer en passant que le scrupule que faisoient les anciens d'user des termes de l'arithmétique en la géométrie ... causoit beaucoup d'obscurité et d'embarras en la façon dont ils s'expliquoient ...* ""

Structure of a Euclidean or Apollonian proposition

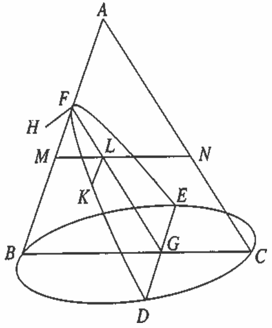
- According to the much later commentator Proclus, there are 6 “parts” – *protasis*, [diagram and] *ekthesis*, *diorismos*, *kataskeuē*, *apodeixis*, *sumperasma*
- The supremely complex and convoluted first sentence two slides ago is the *protasis* of this proposition – the “statement”
- The *ekthesis* then “sets out” the statement by means of a figure and the sort of labeling of important points with letters familiar from school geometry (a *lifesaver* for the reader!)
- The *diorismos* (usually signalled by the phrase “I say that ... " *legō hoti*) then “distinguishes” what is to be shown.

Ekthesis of Proposition 11, beginning (condensed translation following Taliaferro)



Let A be the vertex, ΔABC the axial triangle, and let the other plane cut the plane of the base in DE perpendicular to BC so that FG is *parallel to* AC. The section is the curve DFE.

Conclusion of *ekthesis* and *diorismos* of Proposition 11



Let H be "contrived so that"

$$(*) \quad sq.BC : rect.BA, AC :: HF : FA$$

Finally let K be taken at random on the section and let KL be parallel to DE.

I say that $sq.KL = rect.HF, FL$.

A word on terminology and notation

- The notation here is Taliaferro's modern attempt to capture Apollonius in a (more) readable way
- The Greek is highly conventionalized and abbreviated
- Here $sq.XY$ means (the area of) the square with side XY (Apollonius just says *to apo* XY: literally "the from XY")
- $rect.XY, YZ$ stands for (the area of) the rectangle with sides XY and YZ (*to upo* XYZ: literally "the by XYZ")
- The $:$ and $::$ are standard notation for comparing proportions that may be familiar from analogies

Historical comments

- This was *certainly not discovered by Apollonius*
- Closely related facts used in Archimedes' *Quadrature of the Parabola*, for instance, *without proof*. Archimedes gives a vague reference, probably the lost Euclid *Conics*
- New in Apollonius–use of this property to *define* a parabola, *and* the names we use for conic sections.
- Greek mathematical terminology often "borrowed" common words and gave them special meanings:
    - *parabolē* – noun: a "throwing alongside," comparison, juxtaposition
    - *huperbolē* – noun: a "throwing beyond," excess, superiority
    - *elleipō* – verb: to fall short, be in want of, lack

What's in a word?

- The Greek phrase Apollonius uses for the segments *KL* before deserves special consideration: "*tetagmenōs* to the diameter" – literally means something like *"lined-up"* or *"in order"*, or perhaps even "drawn *in an orderly fashion*" (from the section to its diameter)
- OTOH, many standard English translations of Apollonius (e.g. Heath, Taliaferro, ...) say those parallels have been drawn *"ordinatewise"* to the diameter.
- Interestingly enough, the entry in the standard LSJ Greek lexicon for *tetagmenōs* gives the common meaning and then "ordinatewise" with a reference to Definition 4 in Book I of Apollonius(!) My guess: some mathematical historian *(maybe T.L.Heath?)* provided this citation to the compilers of the lexicon(!)

tetagmenōs to "ordinatewise?"

- First Latin translation of Apollonius to circulate widely in Western Europe by Federigo Commandino (1509-1575 CE); then several others too, including one by E. Halley (1656-1742 CE).
- Commandino's Latin rendering: *ordinatim applicatae* – pretty literal version of the *everyday Greek meaning of the word – "applied in an orderly fashion."*
- Halley (also Latin) has something equivalent; later uses the word *abscissae* for distances along the segments "cut off" by the diameter and the section.
- Note slightly old-fashioned analytic geometry terminology: "abscissas and ordinates" are x and y coordinates(!) Was Apollonius was thinking in coordinate terms after all??

Apollonius' "equation of a parabola?"

- Recall the *diorismos* of Apollonius' Proposition 11: "I say that $sq.KL = rect.HF, FL$."
- Segments like *KL* are said to be drawn "ordinatewise" (mis?)reading Commandino
- If we write the "ordinates" $y = \overline{KL}$ and "abscissas" $x = \overline{FL}$, then noting that *HF* is a fixed segment of length *c*, say, we get the "sideways" parabola $y^2 = cx$ (and *c* corresponds to the length *HF* – called the *orthia pleura* or "upright side" in Apollonius – "latus rectum" later)
- Can get analogous statements for the hyperbola and ellipse as well!

Apollonius "interpreted" for modern mathematicians

- C. Boyer (1906-1976) "*The work of Apollonius in many respects approaches so closely to the modern form of treatment that it not infrequently has been regarded as constituting analytic geometry*."
- H. Zeuthen, *Die Lehre von den Kegelschnitten im Altertum*, first ed. 1886
- T.L. Heath's version of Apollonius, first ed. 1896 – "*so entirely remodelled by the aid of accepted modern notation as to be thoroughly readable by any competent mathematician*" since it "*does not essentially differ from ... modern analytic geometry except that in Apollonius geometrical operations take the place of algebraical calculations*"

A contrary view

- S. Unguru (1931-present), "On the need to rewrite the history of Greek mathematics," Archive for History of Exact Sciences 15 (1975/76), 67–114 (the source of my introductory quote!), a later book with M. Fried on Apollonius
- Unguru's main point: it's geometry pure and simple; Greek mathematics did not have any of the apparatus of symbolic algebra or coordinates
- "Explaining" Apollonius this way (and other similar "reconstructions" of Greek mathematics using modern concepts from the 19th and early 20th centuries) is *perniciously wrong* from the historical point of view.
- *Conceptual anachronism* or "Whig history" – presents the past as leading inevitably to the present

What can we learn from this?

- It points out a fundamental difference between doing mathematics and doing history of mathematics (as history)
- Recognizing logical connections between old and new work and making reinterpretations is a part of what mathematicians do.
- When apparently different things are logically the same, just expressed in different ways, mathematicians can and do treat them as the same(!)
- And we are always looking for those equivalences–finding them can represent an advance in our understanding!

And maybe Unguru had a (small) point?

- As Unguru insinuated in his own nasty way, Zeuthen, his “flame war opponents” van der Waerden, Freudenthal, Weil, etc. were certainly all primarily mathematicians who had eminent research records and then turned to writing history later in their professional careers
- Not surprising that they had the “habits of mind” and point of view of working mathematicians, not historians!
- In particular, to put words in their mouths: “if it’s logically equivalent to a coordinate equation of a parabola, but expressed in geometric terms, then it’s still essentially a coordinate equation”

The "take-home" message

- For intellectual historians, not so much logical equivalences that matter–it's particular features, differences! Each culture, era, scientific school, etc. is a unique and separate thing
- Unguru: The mathematical historian's first and most important job is to understand a body of mathematical work on its own terms, *not on our terms*
- A fundamentally different way of thinking
- The title of a recent article by K. Saito: "Mathematical Reconstructions Out, Textual Studies In" summarizes what's up in mathematical historiography these days!

To be clear ...

- In a recent article, “Apollonius, Davidoff, Rorty, and Zeuthen: From A to Z, what else is there?” (Sudhoffs Archiv, 91 (2007), 1 - 19), Unguru and Fried make it clearer that their “issues” concern Zeuthen’s work as *history*, not as mathematics
- and they contrast Zeuthen’s well-intentioned and mathematically astute (mis)reading with a parodied “post-modern,” deconstructionist view that would deny *any* intrinsic meaning in a text
- Make their point via a (hilarious, fictional) “sexual politics reading” of Apollonius. (Recall the *orthia pleura*? It’s *phallic*; you get the idea!)

Conclusion

- I think the standard histories of mathematics still seriously misrepresent a lot of the history of the subject and that does a disservice to teachers and students!
- I want to try to participate in an active way and learn more about this past.
- Thanks for your attention and I'd be happy to take questions!